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A NON-LINEAR ANALYSIS OF HYDROGRAPHS
FROM FORESTED WATERSHEDS

A THESIS

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A NON-LINEAR ANALYSIS OF HYDROGRAPHS

FROM FORESTED WATERSHEDS

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

SUMMARY

Until recently a basic assumption in methods of analysis and synthesis of storm hydrographs has been that the watershed was a linear system. Hydrologists have long realized that the various flow phenomena which occur in the watershed and which determine the hydrograph shape are non-linear. However, the complexity of non-linear methods of analysis, as well as other factors, has hampered the development of non-linear hydrograph techniques. Lately, however, non-linear methods have begun to draw attention. W. M. Snyder, W. C. Mills, and J. C. Stephens have developed a non-linear technique which takes into account non-linearity within a storm event, as well as differences in response between storms. This technique, which retains the operation of superposition associated with linear processes, has been used successfully on Coastal Plain watersheds. The usefulness of this technique is further exemplified by this study of some forested watersheds of the Tennessee Valley.

Non-linear responses are shown to exist to some extent for all observed events. Responses are not only non-linear but also erratic. The lack of consistency of results makes correlation of the results with watershed characteristics difficult with limited data. However, a few noteworthy conclusions can be made.

The traditional time-area curves derived from topographic maps are shown to differ from the characteristic functions derived from the observed hydrographs. The result of lower unit-hydrograph peak flows for lower intensities of rainfall is also shown.

A computer program which finds optimum values of the model parameters did not work entirely satisfactorily in this study. Several failures to produce entirely credible results indicates that the computer program may need to be refined. The need for further data analysis and possible changes in the model is brought out.

CHAPTER I

INTRODUCTION

For many years the basis for analyzing and simulating storm hydrographs has been the assumption that the hydrograph is a linear, time-invariant response of a watershed to the input of storm rainfall. Mathematically, a linear, time-invariant response is a response which can be expressed by a linear differential equation in which time does not appear as a coefficient. In physical terms, a linear response is one in which the output (streamflow) is a simple linear result of the input (rainfall). In a linear system the output from x units of input is x times the output from one unit of input. Time invariance simply means that the input-output relationship does not change over time.

These principles were adopted by Sherman [1932]* when he introduced unit-graph methods. The unit-graph procedures assume that there is a characteristic time distribution of runoff from an incremental rainfall which is dependent only upon watershed characteristics. The assumption of linearity makes it possible to multiply, offset, and add ordinates of a unit graph to obtain other hydrographs. This well known and useful operation, known as superposition, is perhaps the foremost reason that unit-graph methods have been so greatly developed and extensively used in spite of the questionable assumption of linearity.

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*Refers to reference listed in Appendix C.

Hydrologically, this assumption of linearity of response means that all incremental inputs of effective rainfall are collected and passed out of the watershed in a way which is unique and independent of the varying wetness and surface conditions of the watershed and all aspects of the rainfall event itself. Specifically, if a watershed is a linear system, water stored in the watershed must be linearly related to the outflow and the velocity of the flow in the channels must be constant for all flows. These conditions seldom apply to real watersheds. When one considers the variations in velocity of surface flow and channel flow, as well as the probable non-linear relationship between storage and outflow, the concept of linearity of response appears questionable.

In spite of these non-linear phenomena which determine the hydrograph there are some good reasons why linear unit-graph procedures have been accepted and why they indeed work so well. First, before the advent of the high speed digital computer, non-linear approaches would have been extremely difficult if not impossible. Simple unit-graph convolution was tedious enough. Second, perhaps various non-linear phenomena tend to cancel each other and make the response appear linear. For example, increased flow velocities resulting from larger storms may have the effect of increasing the peak flow and reducing the time to peak. Simultaneously, the fact that greater percentages of the total runoff are generated in remote portions of the watershed could effect the shape of the hydrograph in an opposite manner. The net result would be that the unit hydrograph of the larger storm would have nearly the same shape as the unit hydrograph from the smaller one. A third reason for the acceptance of the linear approaches of hydrograph analysis is that non-linearities are absorbed by the procedures used to separate base flow from the total hydrograph and infiltration from the total rainfall. In using unit-graph procedures one is trying to fit a linear response to input and output which has already been altered by non-linear operations. Finally, the degree of non-linearity may be so small that deviations caused by assuming that the response is linear are small when compared with deviations caused by random variations in the rainfall patterns.

Although the traditional linear unit-hydrograph procedures have been successful, hydrologists have expressed the need for a more general response form which could take non-linearities and time variations into account. Mathematical and conceptual models have been developed to express non-linear watershed responses. These have depended on trial and error solutions and have been, to some extent, empirical. What has been lacking is a generalized, non-linear analytical technique. One such analytical tool has been developed by W. M. Snyder et al. [1969]. In their introductory work, the technique was shown to work successfully on agricultural Coastal Plain watersheds.

The purpose of this study is to analyze hydrographs in order to detect and explain occurrences of non-linearity in the responses of watersheds in the southern United States. The ultimate goal is to relate watershed characteristics to values of parameters derived from the analysis.

CHAPTER II

CURRENT DEVELOPMENTS IN HYDROGRAPH ANALYSIS

Linear Models

The development of methods of hydrograph analysis has been influenced by the systems approach being applied to hydrologic problems. However, even before the terminology and philosophy of systems analysis, hydrologists established conceptual models of the watershed. These early models recognized that the watershed mechanism consists of two distinct operations. First the watershed is a transport device. Rainfall occurring on the watershed flows over the surface, into streams, and finally into the main channel and out of the watershed. The watershed intercepts rainfall which is fed into the channels which act as conveyors. Besides functioning as a collection and transport system the watershed is a storage device. Rain falling on the watershed fills the channels and surface retention areas. Streamflow results from depletion from this storage.

Of course, it is physically impossible to separate storage and transport in a real watershed. All water falling on a watershed is both stored and transported. The relative effects that these two phenomena have on the shape of the hydrograph vary during the progress of an event and depend on watershed characteristics. The treating of storage and transport as discrete operations is merely a conceptual and mathematical convenience.

The conversion of the input of storm rainfall to the output of streamflow has been handled by these two model elements -- a storage with a release mechanism that releases water at a rate proportional to the amount in storage and a transport mechanism. These two elements have been put together in various ways to create watershed models. Some of these models are illustrated in Figure 1. These conceptual models have been linear. That is, the rate of the transport has been constant for all flows and the storage has been a linear function of the outflow. These models are well described by van de Leur [1966].

The earliest of these conceptual hydrograph models was developed by Zoch [1936]. In this model Zoch routed an input distributed in time through a single simple reservoir. It is apparent that water going through a watershed passes through many different types of storage. If the storages are linear they may be lumped and their effect represented by a single storage. This is what Zoch did. He routed triangular, rectangular, and elliptical shapes through his single linear storage.

Clark [1945] used much the same model but used an augmented form of a time-area concentration graph of the watershed as the input to the storage. The storage mechanism Clark took to be channel storage. Zoch considered the soil to be the storage reservoir. Clark's model is significant for bringing out the usefulness of routing procedures in developing hydrograph shapes.

Edson's [1951] derivation of the gamme function as a hydrograph form was based upon an exponential storage depletion (linear reservoir) and a parabola. The parabola Edson took to be a reasonable representation of time-area concentration graphs.

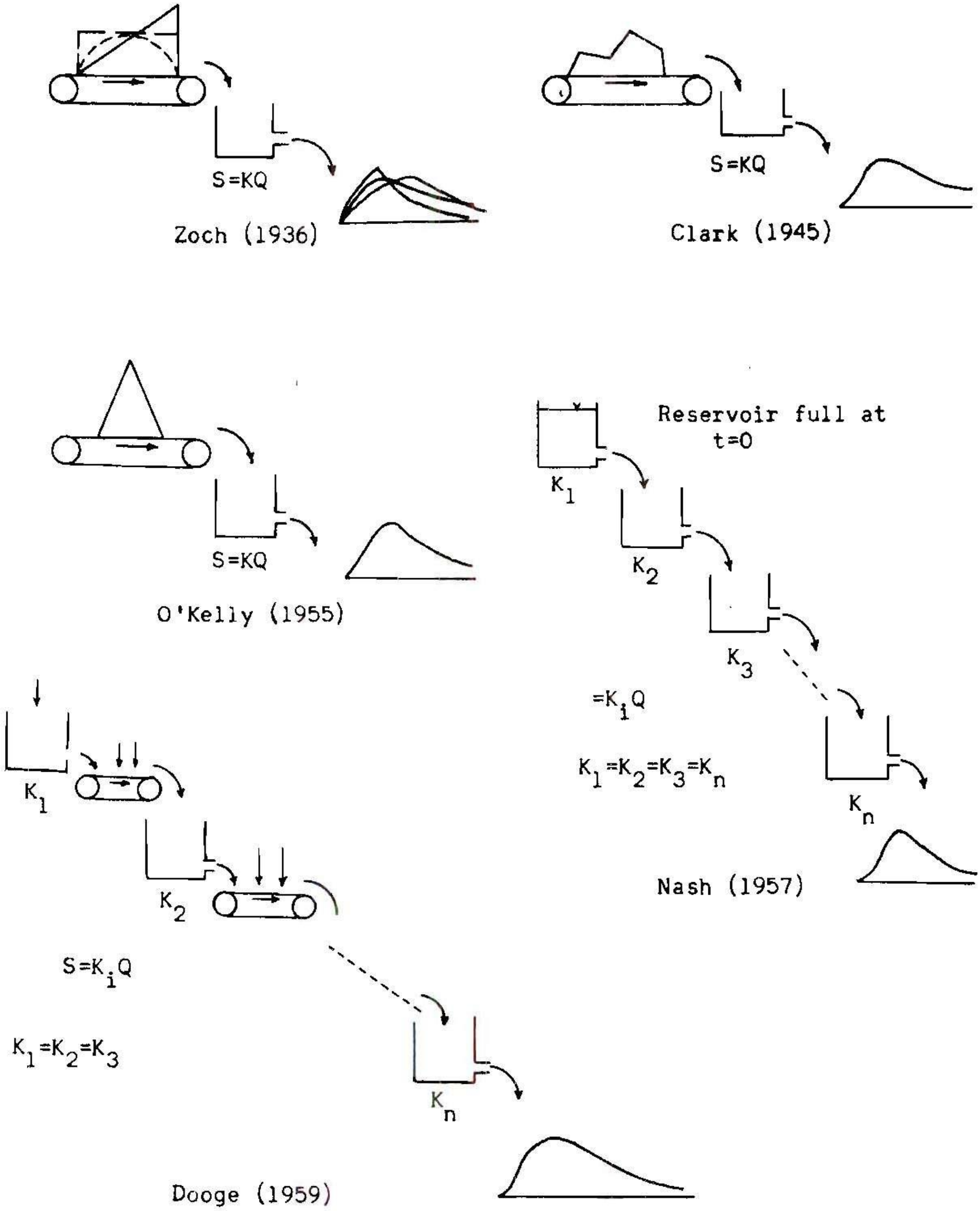


Figure 1. Linear Routing Approaches to Hydrograph Analysis.
(taken from van de Leur [1966] p. 51)

From these relatively simple models Nash [1957] and Dooge [1959] developed models of greater complexity. Nash's model consisted of a cascade of equal linear storages. This new model added another approach to hydrograph routing models. Instead of combining a transport device with a storage, Nash depended only upon the successive storages to determine the hydrograph shape. Dooge added to Nash's model the concept of transport, the linear channel. Dooge's model was more general in that unequal linear reservoirs were allowed. However, Dooge demonstrated that a sufficiently flexible form of unit hydrograph could be obtained by assuming the storages to be equal. Even so, his model contained more degrees of freedom than did either Clark's or Nash's. The input to Dooge's model is the time-area curve.

The concept of deriving unit hydrographs by routing has been used in its various forms and applied by some hydrologists to find relationships between basin characteristics and the hydrograph form. O'Kelly's [1955] work with Irish drainage basins was a systematic application of a model similar to Clark's. However, in his work O'Kelly used isosceles triangles for input instead of time-area graphs. He found that the smoothing effect of the storage reservoir to be great enough to preclude a more complicated form of input (e.g. the time-area graph). O'Kelly related basin size, shape, and slope to the time base of the input triangle and to the storage coefficient.

Sato and Mikawa and Diskin [van de Luer, 1966] have developed models using parallel cascades of linear storages. These complex models attempted to consider non-linearities; however, these were still basically linear approaches.

No matter how complicated a model system of reservoirs one devises, if no allowance is made to cope with non-linear situations, the model is limited. Complex linear models, however refined, are of little use in analyzing or describing a non-linear phenomenon.

Non-linear Models

Because of the difficulties mentioned earlier, non-linear analysis and synthesis have lagged behind the easier linear techniques. Recently, however, some progress has been made in non-linear analysis techniques.

Minshall [1960] has derived a correlation between rainfall intensities and peak flows and times to peak of unit hydrographs. In his study of Illinois basins, Minshall considered the watershed response to remain constant during a runoff event. The variation in response between storm events was the subject of his study. Minshall presented his results in a form which could be readily used by engineers using unit-graph procedures. The significance of his work was the recognition that the watershed response is not constant from storm to storm. His study indicated that unit-hydrograph peak flows increase with rainfall intensity and that times to peak decrease as rainfall intensity increases.

Amorocho and Orlob [1961] pointed out that linear systems are represented by a single convolution integral which is simply the first term of a polynomial series of convolution integrals. The second and succeeding terms of higher dimension account for interaction between input elements. Amorocho utilized the first three terms of the series in analyzing data from a model watershed.

Singh [1964] presented a non-linear model in which the time-area curve was routed through two linear reservoirs. The first reservoir he considered to represent overland flow storage. This type of storage Singh assumed to be unchanging and so the equation which represented this storage was made constant for all storms. The second storage represented channel storage and Singh let the equation which represented this storage vary from storm to storm. In addition he let the time base of the time-area curve vary between events. That is, the time rate of the input to the storage was made to vary. By trial and error, Singh found values for the equations and the rate of input to the storage which gave the best fits of the observed hydrographs. His results were similar to those of Minshall. He found that unit-hydrograph peak flows generally increase with rainfall intensity and that times to peak decrease as rainfall intensity increases.

Prasad [1967] used an analog computer to develop a sophisticated non-linear model. In all past models the storage equation

$$S = KQ^n \tag{1}$$

in which S is the volume of water stored at any time, Q is the instantaneous rate of outflow, and K and n are constants, n has been unity. The equation was linear. One way to accommodate non-linearities is to simply let K vary while n equals one. This is what most investigators have done. Prasad, however, let n be a constant other than unity and also let K vary from storm to storm. In addition to permitting a non-linear relationship between storage and outflow, Prasad expanded the storage equation to take into account unsteady flow. The equation he used was

$$S = K_1 Q^n + K_2 \frac{dq}{dt} . \qquad (2)$$

He felt that the rainfall pattern should have some effect on the storage coefficients so he permitted K_1 and K_2 to vary from storm to storm. Prasad deviated from the customary use of the time-area curve as input and used simply the rainfall excess. The effects of the area and shape of the watershed upon the shape of the hydrograph are reflected in the values of K_1, K_2, and n. Prasad applied his model to hydrographs from seven Illinois basins. In addition to obtaining good fits of the observed hydrographs, he was able to correlate K_1, K_2, and n with basin and storm characteristics. Using values from this correlation he demonstrated the usefulness of his approach. Good fits of observed events were obtained using values from the correlation.

Despite the success of these models, there has been a missing link in the methods of non-linear hydrograph analysis. With the exception of Prasad's work, analysis techniques have depended on some assumption of the shape of the input, or routed, function. Also, previous models have not taken into account variations in the response which occur within the storm. Minshall [1960] gave different hydrographs for early-intense storms and late-intense storms. However, the great concern of most hydrologists has been the variation in watershed response from storm to storm. What has generally been lacking is a purely analytical, non-linear approach to study observed hydrographs.

One such analytical technique has been developed by Snyder et al. [1969]. The analysis derives from a storm hydrograph a curve which is called the characteristic curve or characteristic function. This curve is analogous to Clark's time-area curve. The important difference is that Clark's curve was assumptive while the characteristic curve is derived from the input of the streamflow and rainfall. The analysis also derives parameters which determine a storage coefficient which varies throughout the storm. The model was tested by Snyder et al. on ten storms, two from each of five Coastal Plain watersheds. The results from this first trial were encouraging. The need for further work with this model was demonstrated.

CHAPTER III

THE MODEL

The model that Snyder et al. have developed is similar to the model created earlier by Zoch and used by Clark. Like the earlier model, Snyder's model routes a shape that characterizes the watershed through a storage reservoir. This conceptual reservoir converts this characteristic shape into a unit hydrograph. This new model differs from Zoch's model by being non-linear. Also, this model is more purely analytical since the nature of the storage reservoir and the characteristic shape are determined by an optimizing technique to give the best fit of the observed hydrograph. The earlier models have been based upon assumptions about these two elements.

Two-Stage Convolution

The model routes a characteristic curve through a varying linear reservoir to obtain unit hydrographs for each period of effective rainfall. The unit hydrographs are then applied to a rainfall record to create the storm response. This numeric process is known as two-stage convolution.

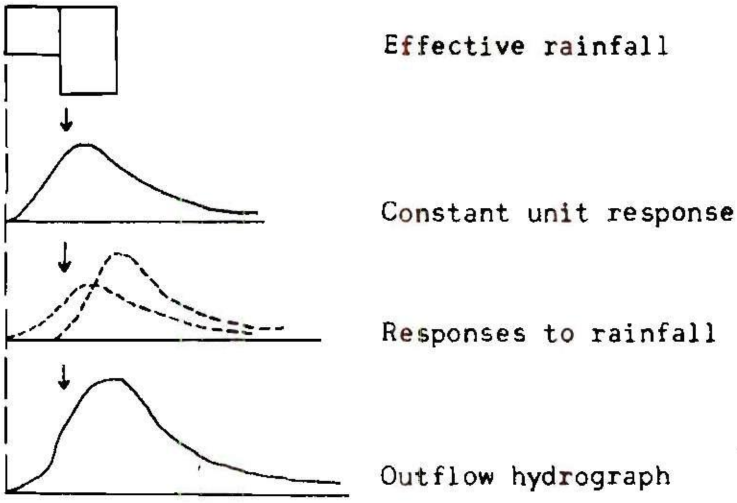
Comparison of One and Two Stages of Convolution

Single-Stage Convolution. Single-stage convolution is well known to hydrologists. It is the traditional process of applying a unit hydrograph to a rainfall record to create a storm response. This process is shown in Figure 2a. Each increment of effective rainfall, shown at the top of Figure 2a, is converted into an equivalent, time-distributed outflow by proportioning the unit linear response. The sum of the time-distributed outflows from all rainfall increments is the total outflow hydrograph. This process of converting all increments of rainfall into the total storm response is single-stage convolution.

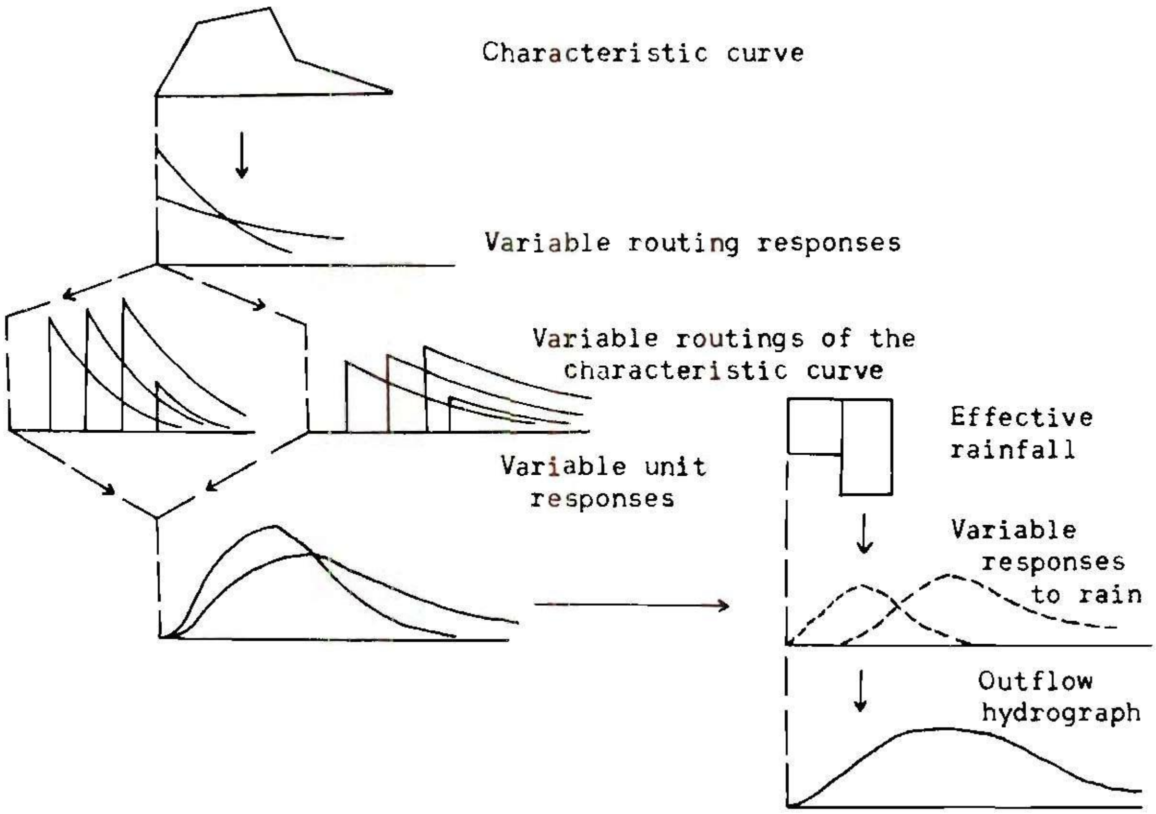
Two-Stage Convolution. Two-stage convolution is obtained simply by adding another "round" of convolution to the single-stage convolution procedure. The additional convolution is used to obtain the unit response, which is then convolved with the rainfall record. Figure 2b shows this two-stage convolution process. At the top of the figure is the characteristic, or core, function. Below the characteristic function in Figure 2b is the routing response curve. The routing response is convolved with ordinates of the characteristic curve as shown. The characteristic function and the routing response function are analogous to the rainfall record and the unit hydrograph, respectively, in the single-stage convolution case (Figure 2a). The second stage of two-stage convolution is the build-up of the storm response from the rainfall record and the various unit responses. This second stage, shown in the lower right-hand portion of Figure 2b, is the same process as single-stage convolution (Figure 2a).

Volumetric Continuity

Any scheme for generating unit hydrographs must meet requirements of physical continuity. This means that output must equal input. For unit hydrographs continuity means that the area enclosed by the unit of output (the unit hydrograph) must represent a volume of water equal to a unit of input which is one inch of rain over the area of the watershed.



(a) Single-stage Convolution



(b) Two Stage Convolution

Figure 2. One and Two Stage Convolution.

Since this unit of input is constant, all of the unit hydrographs must enclose the same area.

This two-stage convolution technique operates so that all of the unit hydrographs enclose the same area. Since convolution is a sort of multiplication process, the area of the resultant curve is the product of the areas of the two convolved curves. When a convolution is performed with a response of unit area, the resultant curve has the same area as the core curve. The form of the routing response prescribes that the area enclosed by the routing curve is always unity. Thus, the responses derived by the first stage of the two-stage convolution are all equal in area to the characteristic curve. The area under the characteristic curve represents the unit volume of input. Therefore, the areas of the unit responses are all equivalent to the unit of input.

Convolution Analogous to Reservoir Routing

Convolution is a numeric process which produces the same form of hydrograph as does the single linear reservoir discussed in the preceding chapter. The first stage of two-stage convolution is the process of obtaining hydrographs by routing. Applying the routing response, which represents outflow from a linear reservoir, to each ordinate of the characteristic curve as shown in Figure 2b is analogous to routing each segment of the characteristic curve through a separate reservoir. Discrete convolution is analogous to collecting the outflows from these separate reservoirs.

It should be noted that all ordinates of the characteristic curve are convolved with only one routing response to obtain any single unit hydrograph. Conceptually, this means that all portions of the characteristic curve are routed through identical reservoirs for any single storm period. Since the reservoirs are linear and equal, they may be combined. In other words, the several linear reservoirs are equivalent to a single reservoir.

It should be explained that although the derivation of each unit response is accomplished by convolution, a linear process, the overall technique is non-linear since the character of the routing reservoir is allowed to vary during the rainfall event.

The Routing Function

The Storage Equation

The reservoir through which the characteristic function is routed is defined by the equation

$$S = KQ \tag{3}$$

which is the linear special case of Equation (1). Since flow, Q, can be expressed as the rate of change of storage, then

$$-S = K \frac{dS}{dt} . \tag{4}$$

Since the reservoirs are fed instantaneously, Equation (4) is the complete equation of flow from the reservoirs. Integrating Equation (4) yields

$$\frac{S}{S_o} = e^{-t/K} \tag{5}$$

and differentiating this gives

$$-\frac{dS}{dt} = Q = S_o \frac{1}{K} e^{-t/K} . \tag{6}$$

If the instantaneous input, S_o, is unity, then

$$q = \frac{1}{K} e^{-t/K} \tag{7}$$

in which q is an ordinate of the exponential decay function at time, t, and K is the storage coefficient.

Equation (7) represents the routing response discussed in the preceding section. Notice that the integral of Equation (7) between the limits of zero and infinity is equal to unity regardless to the value of the storage coefficient, K.

The Reservoir Coefficient

In past hydrograph routing schemes, K in Equations (3) through (7) has been taken as a constant for the entire storm event and for all conditions of flow. A constant storage coefficient in Equation (3) indicates a linear relationship between storage and outflow. However, it has been shown [Horton, 1937] that storage and flow are not necessarily linearly related. A more flexible form of equation which could take into account non-linearity of the storage-outflow relationship has been needed.

One flexible form to relate storage and flow was that developed by Prasad (see Equations (1) and (2)). Snyder et al. have used another form. This latter form has been used in this study.

In this study the storage coefficient is made variable between time increments. This coefficient is determined for each period of rainfall by the equation

$$K = \frac{1}{K_1 + K_2 Q + K_3 \frac{dQ}{dt}} \tag{8}$$

in which K_1, K_2, and K_3 are constants for any given storm. Q and $\frac{dQ}{dt}$ are flow and change in flow measured at the beginning of the time increment for which K is computed. Values for K_1, K_2, and K_3 are computed by the optimizing technique to give the best fit of the observed hydrograph.

The Characteristic Curve

The other model component which determines the unit hydrographs is the characteristic curve (see Figure 2b). Qualitatively, the characteristic curve represents some sort of time collection of runoff and is determined by watershed characteristics. However, at this time it is impossible to predict precisely the shape of the characteristic curve.

It is desirable to use some flexible form to represent the characteristic curve. The actual shape of the characteristic function as determined by watershed characteristics might be quite complex and not well defined by simple geometric forms. The function chosen to represent the characteristic curve should be able to approximate these complex forms.

The linear-segmented form is one such flexible form which has been used in hydrologic investigations [Snyder, 1967]. A linear-segmented form with five valued, or "angle," points was used in this model. The values of points between angle points are derived by linear interpolation between angle points.

Technique of Optimization

The technique employed to find values of parameters of the characteristic curve and routing function is a combination of components regression and non-linear least squares. The technique is an iterative process in which successive rounds of corrections are made to the analysis parameters. Corrections are computed after each iteration to improve the fit of the re-created event to the observed event.

Normally, the corrections computed by the optimizing technique become insignificant after a few iterations. The term convergent refers to these corrections becoming negligible after a finite number of iterations. Convergence implies that the parameters have reached stable values which give the best fit of the observed hydrograph. Non-convergence is the condition in which the corrections do not become insignificant after a number of iterations.

A more detailed discussion of the optimizing technique is outside the scope of this paper. A complete discussion of this technique has been given by DeCoursey and Snyder [1969].

CHAPTER IV

DATA

The Watersheds

Criteria

It is customary and prudent to place limitations on the types of watersheds used in a hydrograph study. To select data indiscriminately from many types of watersheds could cause meaningful results to be obscured by variations caused by differences in the watersheds. Consequently, for this study certain criteria were established to select watersheds which would limit the variability of watershed characteristics.

One principal criterion was that the watersheds studied be forested. Restricting the study to forested areas only should suppress any differences in watershed responses usually caused by differences in land cover. Also, the surface conditions of forested areas are likely to remain fairly constant for long periods of time. Whatever effect surface conditions --litter, soil texture, etc.-- would have on the watershed response would be the same for storms separated in time.

The study was further restricted to watersheds in the southeastern United States. This area was selected because it is an area of fairly uniform climate and because data were readily available for watersheds in this area.

Location

Four watersheds were selected to be used in the study. All lie within the Tennessee River basin and are administered by the Tennessee Valley Authority. Two of the watersheds lie in Alabama; the others are located in Tennessee. The four watersheds are representative of three geological provinces in the southeastern United States.

Adequate streamflow and rainfall records were available for all watersheds. White Hollow and Pine Tree Branch watersheds, which have been involved in reforestation projects, have long records available. The other watersheds have just recently been instrumented and therefore do not have as much available data as do the other two.

White Hollow Watershed

White Hollow watershed is a small watershed which lies between the Powell River and Clinch River branches of Norris Reservoir in northeastern Tennessee. The basin lies in the Valley and Ridge province of Tennessee, and as can be seen from Figure 3, the watershed consists of a series of nearly parallel valleys and steep ridges. The watershed is 1715 acres in size. The average slope of the watershed is 30 per cent. The watershed varies in elevation from 1680 feet to 1080 feet above sea level. The channel slopes average 300 feet per mile. The soil is principally Fullerton or Clarksville cherty silt loams which are moderately permeable. The soil is underlain by a porous limestone deposit. Consequently, interflow is a very prominent phenomenon in the watershed. Many springs are known to exist in the basin. The average annual rainfall is 47 inches. The present land cover is a mixture of species of hardwoods with some pine trees.

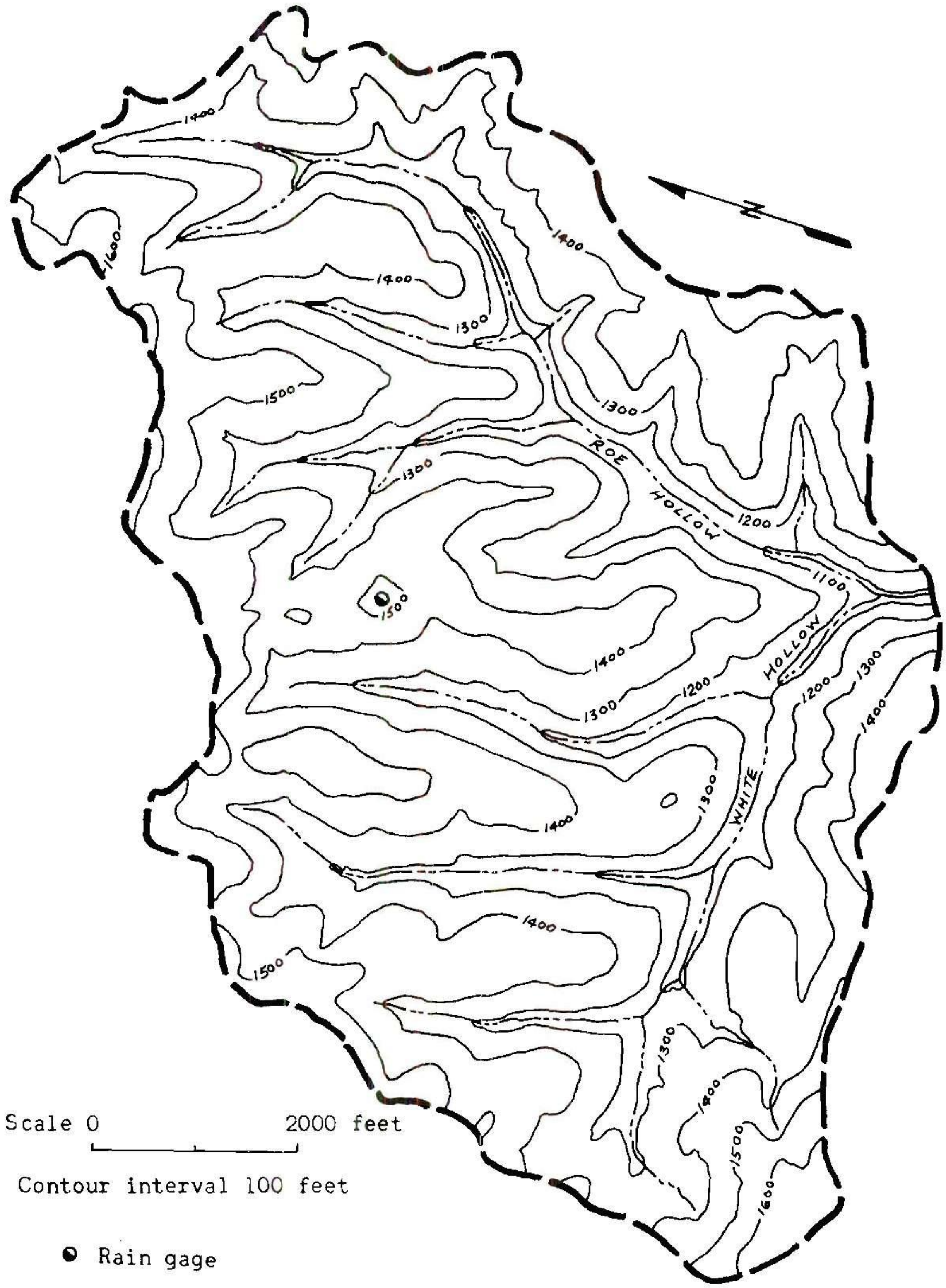


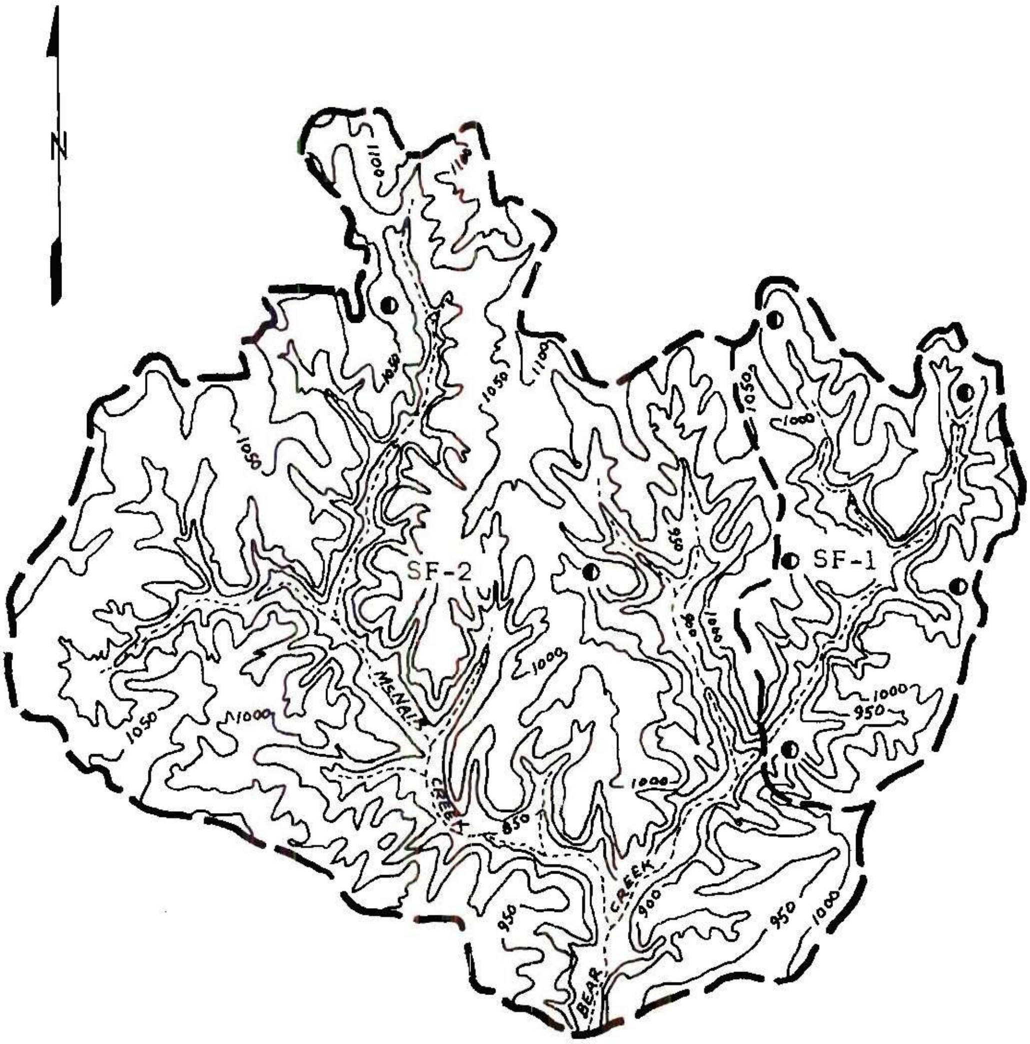
Figure 3. The White Hollow Watershed.

Bear Creek Watersheds

Two watersheds in the TVA Upper Bear Creek Experimental Project [TVA, 1966], designated SF-1 and SF-2, were used. These two watersheds, 2.48 and 13.9 square miles in size respectively, are located in northwestern Alabama. Figure 4 shows the topography of these watersheds. The highest point in these two watersheds is 1115 feet. The lowest point, the gage of SF-2, is 929 feet above sea level. The average channel slope of SF-1 is approximately 60 feet per mile. The channel slope of SF-2 is approximately 35 feet per mile. The soils on the rather flat ridge tops of these watersheds are derived from the upper Coastal Plain formation. These soils are moderately permeable sandy loams and sandy clay loams. On the steep slopes descending from the ridgetops to the drainageways, another type of soil is encountered. Thin, sandy soil derived from shale and sandstone parent materials is found on these slopes.

Pine Tree Branch Watershed

Pine Tree Branch watershed, shown in Figure 5, was the fourth watershed involved in this study. This watershed covers 88.2 acres in western Tennessee. Pine Tree Branch is a tributary of the Beech River which flows eastward into the Tennessee River. The watershed lies in an area which is geologically a part of the Mississippi Embayment. The soil is a rather permeable sandy loam. The watershed lies between 595 and 450 feet elevation, and the main channel slopes from two to ten per cent. The cover is principally pine forest planted in 1946.



Scale 0 $\frac{1}{2}$ 1 mile

◐ Rain gages

Contour interval 50 feet

Figure 4. The Bear Creek Watersheds.

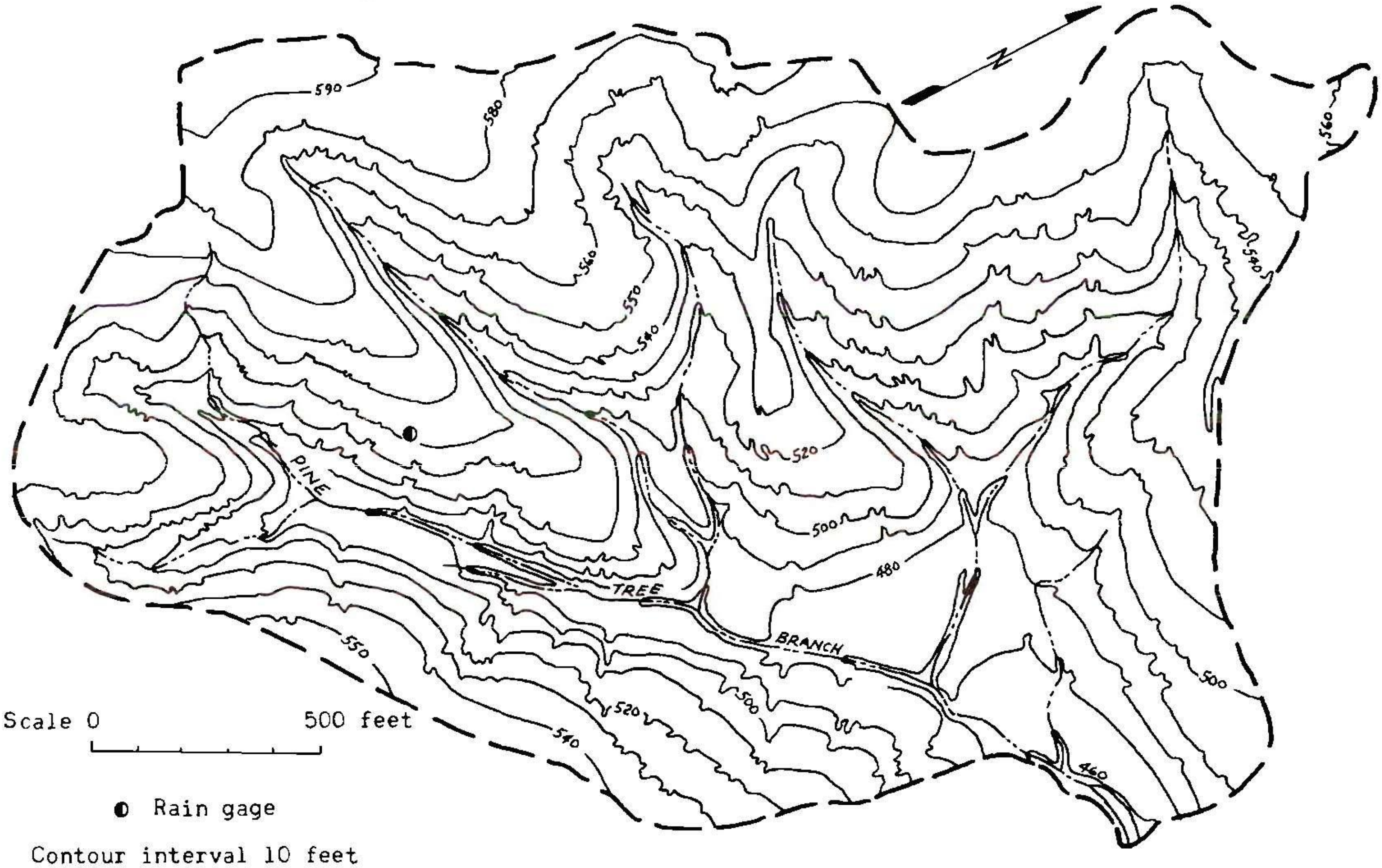


Figure 5. The Pine Tree Branch Watershed.

Storms

General Criteria

Just as it is wise to restrict the types of watersheds that are involved in a hydrograph study, it is also prudent to place restrictions on the types of rainfall events considered. Several standards were established to limit the selection of rainfall events. The purpose of these criteria was to suppress, as far as possible, variations in response caused by differences in storm events and watershed conditions.

Winter Storms. Of course, no two storm events are ever alike, but there are two broad classifications into one of which all events must fall. Rain falling on these watersheds comes from either convective or frontal storms. Convective storms are usually short, have high intensities of rainfall, and are somewhat limited in size. Frontal storms are associated with long durations, low intensities of rainfall, and occur over much larger areas than do convective storms. Frontal storms are more likely to produce an even intensity of rainfall over a watershed at any given instant. For this reason, frontal storms were chosen for this study.

Since frontal storms occur principally in winter months this study was limited to events occurring in the period from October through mid-April. It is sometimes hard to determine from rainfall records just which storms are convective and which are frontal. In the Tennessee Valley area, however, few convective storms occur in the winter. Therefore, it is reasonably certain that the events used in this study are all frontal storms.

Length of Records. Another restriction of the time period from which storms were selected was necessary. Two of the watersheds, Pine Tree Branch and White Hollow, have been undergoing reforestation. The effect that this change in cover has had on the hydrographs from these watersheds has been demonstrated [TVA, 1961; TVA, 1962]. To limit the effects that these changes in cover would have on the results of this study, only the last 15 years of record were used in securing data.

Hydrograph Shape. Rainfall events were also selected according to the shapes of the hydrographs produced. It was not known how well this new technique would work on complex or multiple-peaked hydrographs. Furthermore, it was not in the scope of this study to test the flexibility of this technique to analyze complicated hydrographs. Therefore, with only one exception, this study was limited to storm events which produce simple, single-peaked hydrographs.

The Storm and Flow Data

Twenty-three hydrographs and rainfall records were obtained from the Tennessee Valley Authority for study. Eight were chosen from Pine Tree Branch, six were chosen from White Hollow, and four and five were selected from Bear Creek SF-1 and SF-2 respectively. These storms are listed in Table 1. It should be noted in Table 1 that the selected events are cross sections of different peak flows and storm durations. The data are representative of winter storms for these watersheds.

Time Increments. The choice of time increments, the time interval at which ordinates of the hydrograph and amounts of rainfall are fed into the computer, was usually determined by the form of the data obtained from the TVA. Streamflow abstracts and rainfall histograms were available at one-hour time increments for White Hollow and Pine

Tree Branch watersheds. Consequently, one hour was the time unit generally used as the time increment for these two watersheds. Rainfall and streamflow data from the Bear Creek watersheds, SF-1 and SF-2, were available at 15-minute and 30-minute intervals respectively. However, it was sufficient to use half-hour and one-hour increments in the analysis for these watersheds.

Time Base

The time base of the hydrographs, or the number of hydrograph ordinates, used in the analysis, varied between watersheds and between events on any single watershed. These time bases are listed in Table 1. Several rather qualitative standards were used in determining the hydrograph length to be used in the analysis. Generally, enough ordinates were used so that the remaining tail appeared to be a simple exponential recession. Also enough ordinates were used so that the remaining tail contained only 10 to 15 per cent of the storm flow. Another factor which determined hydrograph length was how well the remaining tail could be estimated. Hydrograph ordinates were selected until a point on the hydrograph was reached from which a straight line could reasonably fit the remainder of the hydrograph.

Events Eliminated

Three of the events were eliminated from the study because of apparent errors in the data. Two storms from Pine Tree Branch were not included in the study because the streamflow records and rainfall records appeared out of phase in time. A storm on Bear Creek SF-1 was eliminated because the total streamflow exceeded the total rainfall.

Table 1. Storm Data

Watershed and Date of Storm	Antecedent Condition	Total Rainfall in Inches	Peak Flow in CFS	Length of Storm in Hours	Length of Hydrograph in Hours
White Hollow					
11-25-64	Dry	1.37	6.1	19	75
10-18-66	Dry	.59	2.7	7	60
12-11-64	Dry	1.32	7.1	13	75
3- 8-56	Wet	1.12	27.4	6	60
12- 7-57	Wet	2.27	42.3	33	60
3- 4-66	Dry	.59	2.7	7	65
Bear Creek SF-1					
4- 7-64	Wet	1.75	265	7	18
2-10-65	Wet	1.80	184	15	23
3-25-64	Very Dry	2.51	133	20	43
3-14-64	Dry	1.72	237	7	25
2-11-64	Wet	1.29	135	4	19
Bear Creek SF-2					
4- 7-64	Wet	1.68	972	8	35
2-10-65	Wet	3.33	848	30	73
3-25-64	Dry	2.39	532	20	60
3-14-64	Dry	3.08	981	22	65
Pine Tree Branch					
3- 8-60	Wet	1.30	.27	7	35
10-27-64	Dry	1.60	.17	4	25
11-17-58	Wet	1.17	.07	11	35
11-13-57	Wet	2.69	10.7	10	50
4-15-58	Wet	1.11	.18	8	35
4-10-62	Very Wet	2.22	26.9	8	70
3-16-63	Very Wet	1.15	1.62	2	35
1-21-59	Wet	1.26	.70	6	30

Pre-Analysis Steps

Two pre-analysis steps that have a great effect on any hydrograph study are base-flow separation and effective-rainfall determination. These two procedures can have a great influence on the outcome of any study. Any process used to determine base flow and effective rainfall should be as hydrologically correct as possible.

Base-Flow Separation

Many hydrograph studies have concentrated on the surface flow hydrograph. To separate what is surface flow from the total flow record, methods have been developed to estimate and subtract the increasing groundwater flow from the storm response. These methods of treating the hydrograph have been widely used and accepted.

Another, more recent approach [Snyder, 1968; Snyder and Curlin 1969], has been developed in which all aspects of the watershed response -- surface runoff, groundwater flow, and interflow -- are considered in the analysis. Base flow, in this context, is the flow that would have occurred had there been no rainfall. This flow is estimated by fitting an exponential recession to the streamflow occurring immediately before the storm. This exponential is extending under the storm response and is subtracted from the total streamflow. The resulting curve is termed the observed hydrograph.

A simplification of this procedure was applied to the data in this study. For the purpose of this study a simple straight line sufficiently represented the actual base-flow recession. The actual recessions for these small watersheds are very nearly horizontal lines. Therefore, in this study, as in the earlier work with this model, the hydrographs studied were derived by merely subtracting the flow at the beginning of the rise of the hydrograph from all ordinates of the streamflow record.

Effective-Rainfall Determination

Effective-rainfall determination is the process of converting the volume of water in the observed hydrograph into amounts of rainfall distributed in time. The process is directed by continuity -- input must equal output. The total amount of rainfall distributed is the depth of water needed to cover the watershed area to create a volume of water equal to the volume of water contained in the hydrograph.

Several techniques have been developed to determine how the rainfall occurring as streamflow is abstracted from the recorded rainfall. These techniques, for the most part, are based upon infiltration capacities. Any application of these techniques should be tempered with a knowledge of the watershed involved. For example, low intensities of rainfall might theoretically be entirely absorbed in the upper layers of soil. However, in a real watershed there are the surfaces of streams and impervious areas which immediately convert any rainfall to streamflow. Watershed characteristics should be carefully considered in applying any method of effective-rainfall determination.

No strict numeric technique was used in this study to determine effective rainfall. However, several general rules were used to guide distribution of effective-rainfall amounts. One principle used was that early in a storm large percentages of rainfall are used to replenish soil moisture. Effective rainfall, the rainfall that makes up the hydrograph, is therefore a smaller percentage of total rainfall during the first periods of a storm than during later periods. The amount of rainfall that does not appear in the hydrograph approaches a constant small amount of each increment of storm rainfall as the event progresses. A third principle relates to the example cited above. For low intensities of rainfall, the rain that falls on impervious areas and on streams appears in the hydrograph. Thus, for even low intensities of rainfall a small percentage of rainfall becomes effective rainfall.

CHAPTER V

RESULTS

There are several criteria which a hydrograph analysis technique should meet in order to be useful to hydrologists. The results from an analysis technique should determine if the technique meets these tests. The overriding criterion is whether the technique provides reasonable results. Reasonable, in this context, means that the results do not contradict basic principles of hydrology. Unit responses should be reasonable unit hydrographs for the watersheds considered. Also, the analysis must show that the technique is capable of re-creating observed events. The closeness of the fit of the observed hydrograph is a measure of the quality of the technique. A third measure of usefulness is whether the technique creates new knowledge. Does the technique provide information that other techniques do not? Does the analysis provide some missing bit of information?

The results of this study reveal that the technique does meet the above criteria. The derived hydrographs are shown to be consistent with the watersheds involved. Good fits of the observed events are obtained. Also, new information about the linearity of the watershed response is revealed.

Direct Results

Unit Hydrographs

Rationality. A visual inspection of the non-linear unit responses indicates that the analysis technique produces reasonable results for the watersheds considered. Most of the unit responses plotted indicate storm runoff patterns which are rational for the watersheds involved. Figure 6 shows unit hydrographs from various storms which are typical for each of the four watersheds. Although there is a great difference in the hydrographs plotted in Figure 6, these unit responses represent flow patterns which could have occurred.

Variations Within Storms. Variations of responses within a storm event indicate that the watershed is responding differently to rainfall inputs. The degree of variability of the unit responses is a direct measure of the non-linearity of the watershed response.

All events showed variations in the unit responses to some degree. Two typical series of unit responses are shown in Figures 7 and 8. Although both series of hydrographs are from the Pine Tree Branch data, they are typical of the types of responses shown by all of the watersheds. It should be noted that the peak flows of the unit hydrographs in Figure 7 increase as the event progresses. The peak flow of the last unit response is about 175 per cent of the peak flow of the first unit response. An opposite trend is shown in Figure 8. The unit peak flows for each response of this event decrease as the event progresses. The final response of this series of hydrographs has a peak flow which is only 25 per cent of the peak flow of the first unit response. These two sequences of unit hydrographs indicate that possibly different phenomena are controlling the shape of the hydrograph in these two cases. At any rate, the two analyzed events represent positive evidence of the non-linearity of the watershed response.

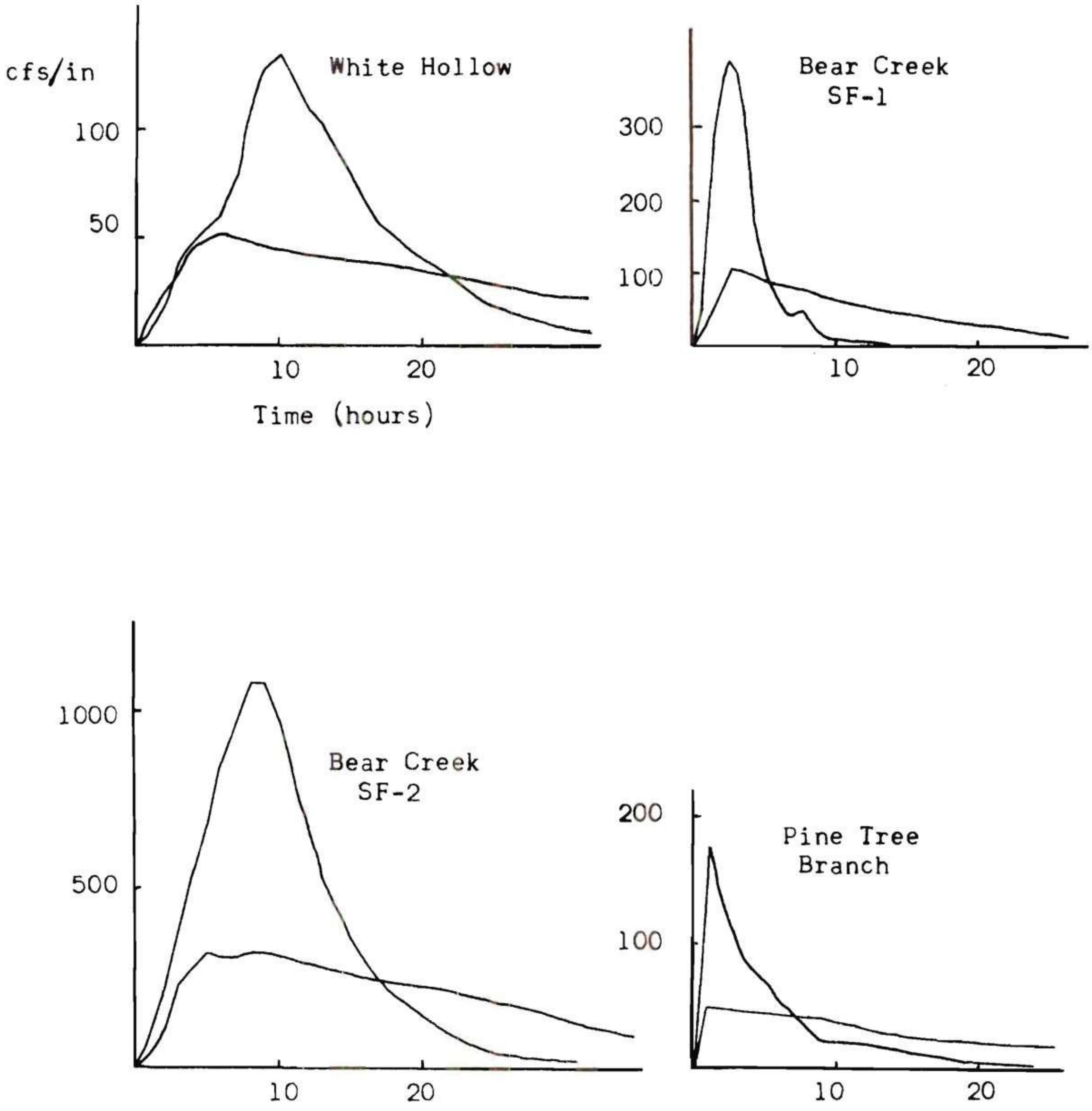


Figure 6. Typical Unit Responses.

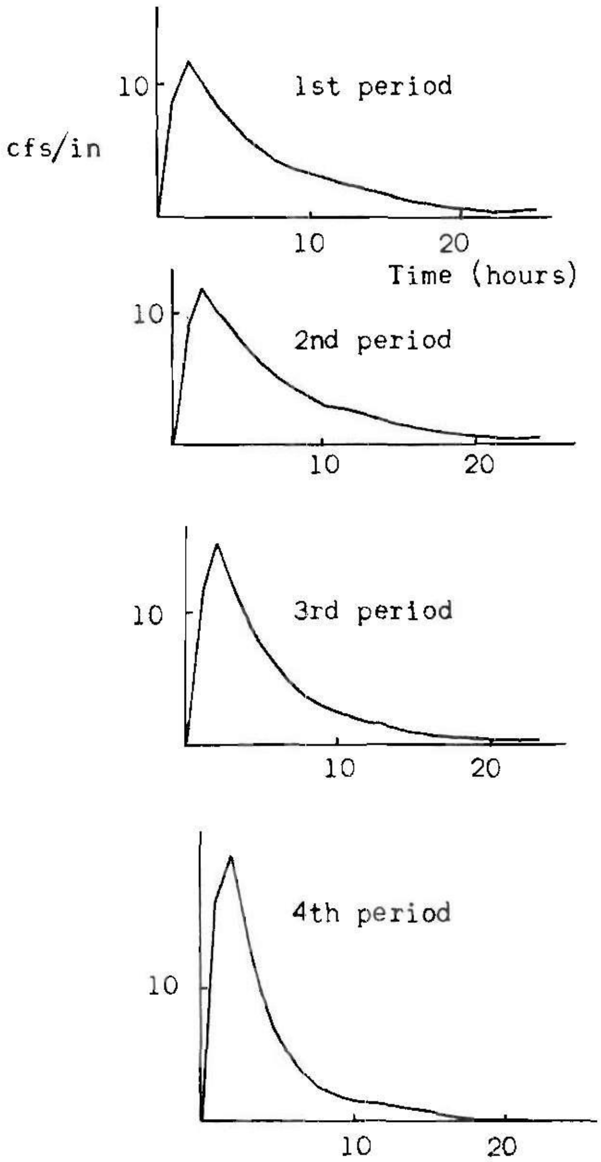


Figure 7. Unit Hydrographs for the Pine Tree Branch Watershed Storm of October 27, 1964.

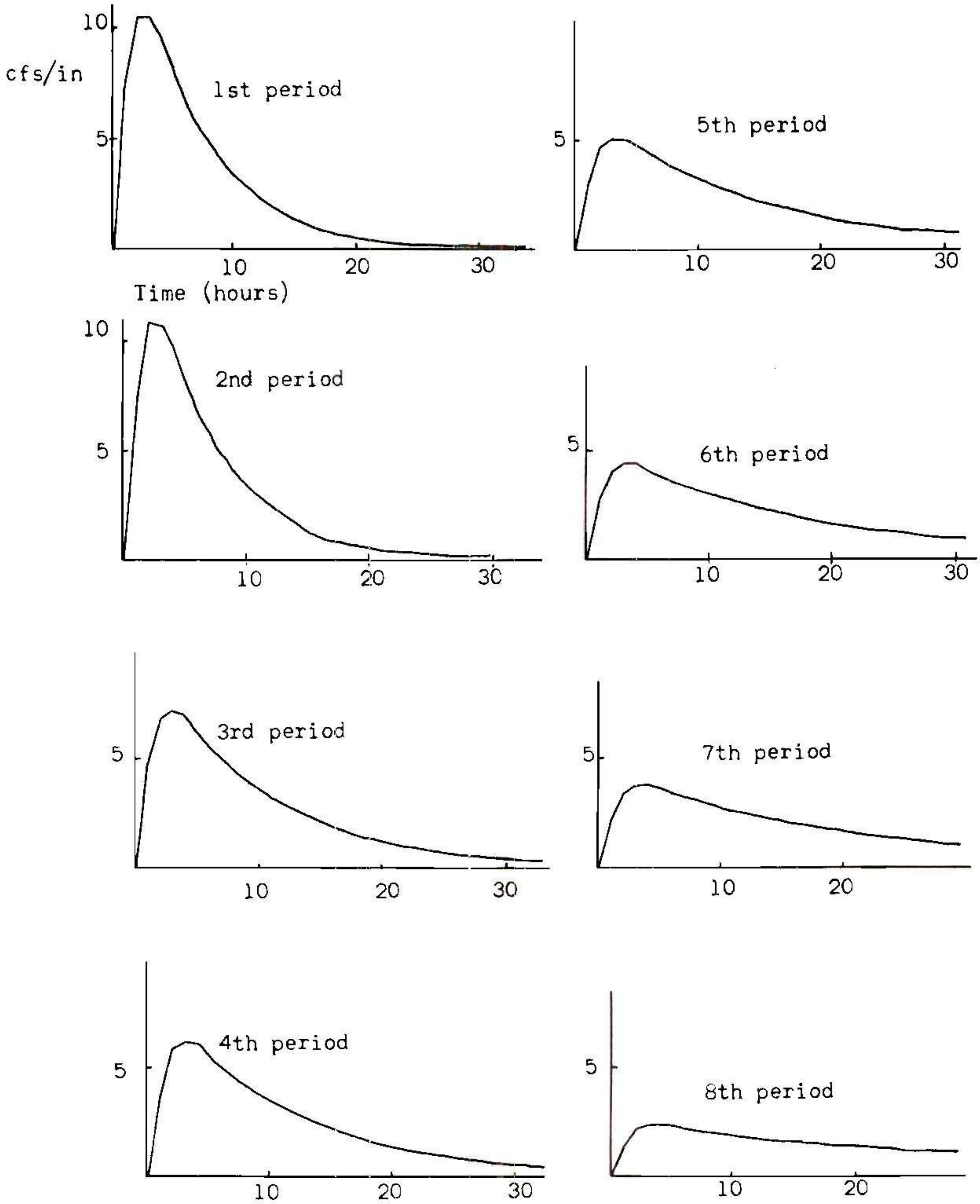


Figure 8. Unit Hydrographs for the Pine Tree Branch Watershed Storm of April 15, 1958.

Fitting Observed Events

An important test of an analysis technique is how well it can re-create observed events. The degree of fit of the observed event is a measure of the flexibility of the technique. The quality of fit also offers evidence that the model could represent the watershed process. A model that will not closely fit an observed event is a model that does not well represent the watershed system.

Hydrographs. Good to excellent fits were obtained for all of the events analyzed. Figure 9 shows typical fits obtained in the study. The best fit is shown at the top of Figure 9. In this example the predicted hydrograph overlaps the observed hydrograph for nearly the whole length of record. The middle hydrographs in Figure 9 represent a degree of fit which is probably most representative of the fits obtained from the events analyzed. In this case the predicted hydrograph peak comes to within 10 per cent of the observed peak. The fit of the observed recession is not as good as that of the top example. The example at the bottom of Figure 9 is one of the relatively poorest fits obtained in the analysis. Yet, this fit is still acceptable by most standards. The predicted peak is less than 5 per cent greater than the observed peak. The predicted time to peak is 18 hours whereas the actual time to peak is 16 hours. The worst feature in this example is the difference in the observed and predicted recessions.

Correlation Coefficients. The correlation coefficient is a statistical measure of the degree of fit. The closer the correlation coefficient is to unity, the better is the fit of the observed hydrograph.

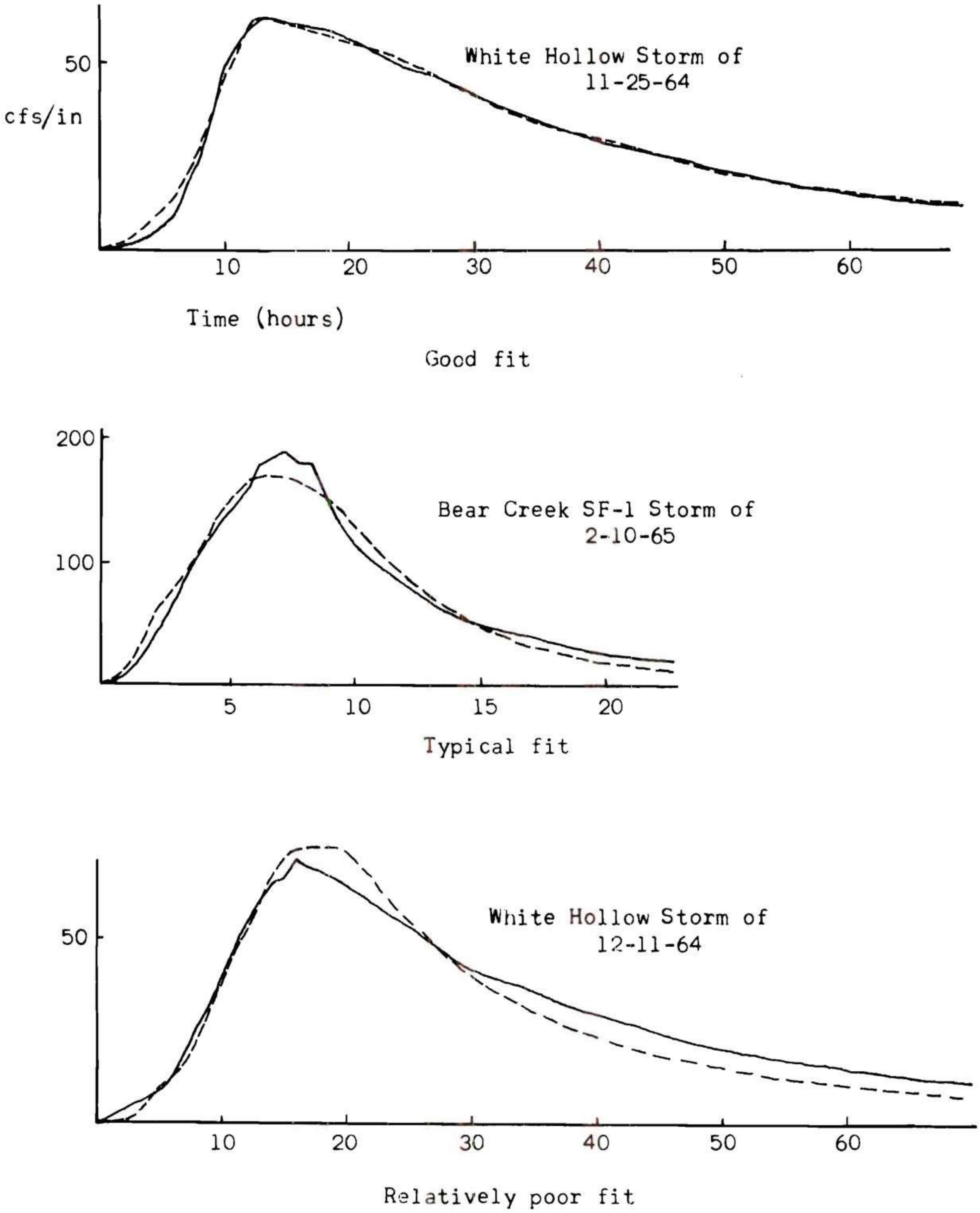


Figure 9. Examples of Fits of Observed Hydrographs.
—— Observed hydrograph -- Predicted hydrograph

The events analyzed produced correlation coefficients close to one. The values of the correlation coefficients for the three examples of Figure 9 are, from top to bottom, 0.998, 0.986, and 0.972. The average correlation coefficient for all events was 0.973.

Characteristic Function

Another facet of the results is the characteristic function. It is assumed that the shape of the characteristic function gives some indication of how areas of the watershed are producing runoff. Consistency in the shapes of characteristic functions from the same watershed for different events is assumed to be an indication of the degree to which the watershed is consistent in runoff production. Some variations in the shapes of the characteristic curve should be expected. In spite of limiting the study to winter storms on forested watersheds, variable factors might still affect the shape of the characteristic curve.

Fairly consistent characteristic functions were obtained for all the watersheds. These characteristic functions are shown in Figure 10. Pine Tree Branch and Bear Creek SF-1 watersheds have the most consistent characteristic functions. The characteristic curves of Bear Creek SF-2 and White Hollow watersheds are somewhat more variable.

The Routing Function

The routing function, which supplies the element of non-linearity in the model, determines the shapes of the unit responses. A measure of the variability of the routing function is also a measure of the variability of the watershed response.

The value of the storage coefficient, K, from the routing equation,

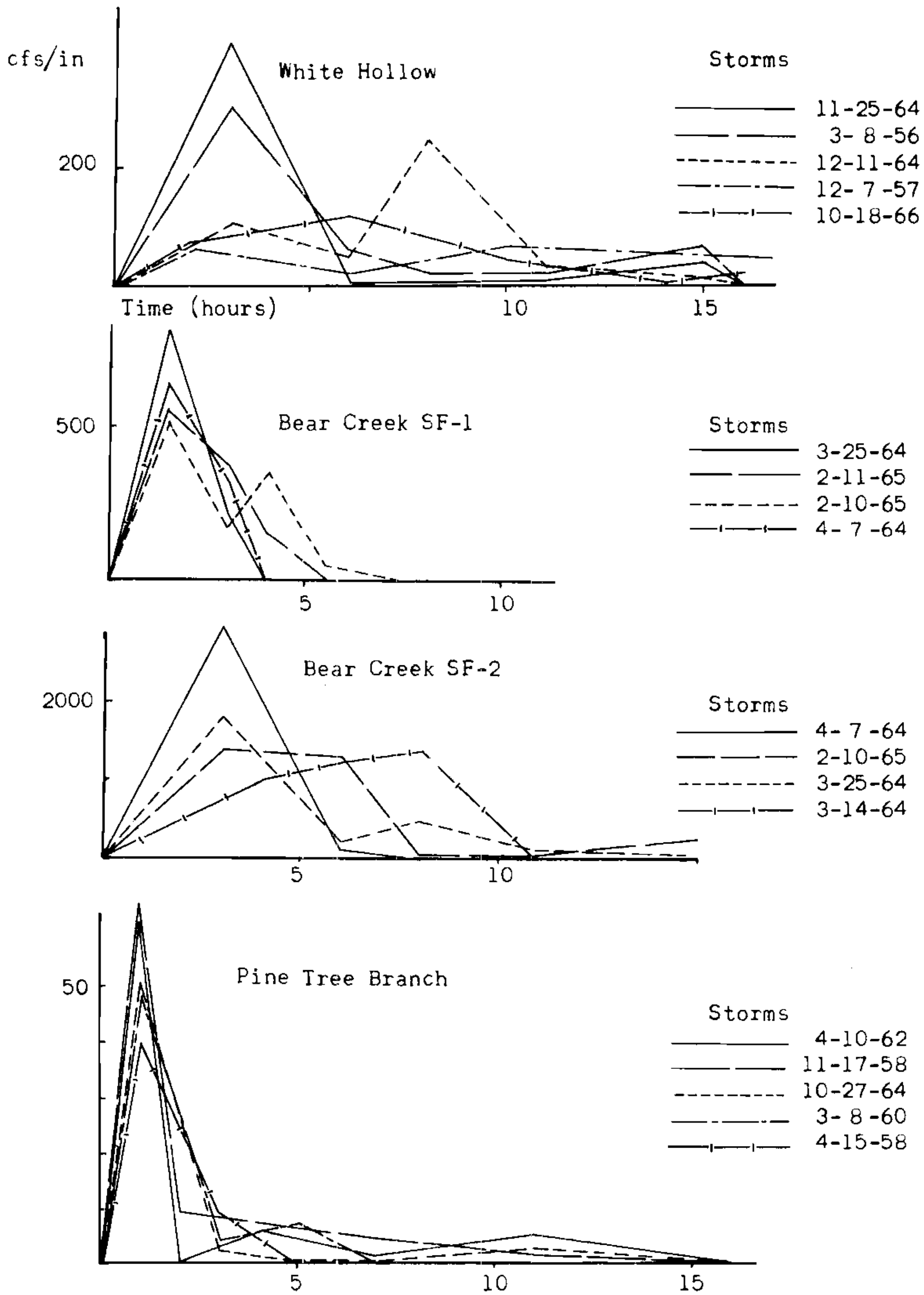


Figure 10. Characteristic Curves.

$$q = \frac{1}{K} e^{-t/K} \tag{7}$$

is a measure of the shape of the routing function. K is measured in time units. It is the distance to the centroid of the area below the routing curve. The value of K is also significant to the routing process. For an instantaneous input, or runoff, routed by Equation (7), half of the input will have passed out of the watershed by K time units.

Values of the storage coefficients from this study reflect a great deal of variation in the responses of the watersheds for the storms studied. Maximum and minimum values of K are given in Table 2.

Interpretation of Results

Simply obtaining data from a new analysis technique may be in itself unimportant. What is important is whether the results can be used to increase understanding of a hydrologic event. Numeric results have little significance unless they can supply new knowledge.

The purpose of an analysis is to find out how and why an observed event occurred. Results of an analysis are useful if some relationships can be found to account for what has been observed. In a hydrograph study such as this, the relationship between watershed characteristics and the shape of the hydrograph is the relationship investigated.

In this research, statistical correlations of watershed characteristics with analysis parameters is not prudent because of limited data and because of limited experience with the model. However, finding factors which effect the shapes of the characteristic curves and routing

Table 2. Maximum and Minimum Values of the Routing Coefficient

Watershed	Date of Storm	Maximum K in Hours	Minimum K in Hours
White Hollow	11-25-64	599.0	26.4
	12-11-64	158.7	6.7
	3- 8-56	32.5	13.6
	12- 7-57	31.4	2.2
Bear Creek SF-1	2-10-65	7.6	3.7
	2-11-65	6.3	2.9
	4- 7-64	3.6	1.3
	3-25-64	55.0	2.3
Bear Creek SF-2	3-25-64	47.0	1.7
	3-14-64	121.8	4.6
	2-10-65	35.0	4.2
	4- 7-64	8.7	2.2
Pine Tree Branch	10-27-64	5.6	2.7
	3- 8-60	37.2	5.9
	4-10-62	3.1	0.7
	11-17-58	11.7	6.2
	4-15-58	31.6	5.7

curves and finding trends in unit-response variations are matters which should be considered.

The Characteristic Curve

There should be some relationship between the characteristic curve and the shape of the watershed. As mentioned in Chapter II, several linear models have been based upon routing time-area curves through linear reservoirs to create unit hydrographs. Obtaining hydrographs in this way is not new, and does have merit.

The prime question now is how similar are the time-area curves and the statistically derived characteristic curves. Earlier investigators [Clark, 1945; Singh, 1964] have shown that routing the time-area curves does give good unit hydrographs. However, are the time-area curves which give good hydrographs similar to the statistically derived characteristic curves which supposedly give the best hydrographs?

Comparisons were made of the derived characteristic curves and the time-area curves of the watersheds. These two curves for each watershed are shown in Figure 11. The method of determining the time-area curves is discussed in Appendix A.

The characteristic curves shown in Figure 11 are not similar to the time-area curves. The best agreement between these two curves appears with the Bear Creek SF-2 watershed. The two corresponding curves for the other three watersheds are quite dissimilar.

Variations in Response

One purpose of this study was to find what effects rainfall intensities have on the shapes of the unit hydrographs. Several studies [Minshall, 1960; Singh, 1964; Prasad, 1967] have shown that unit-

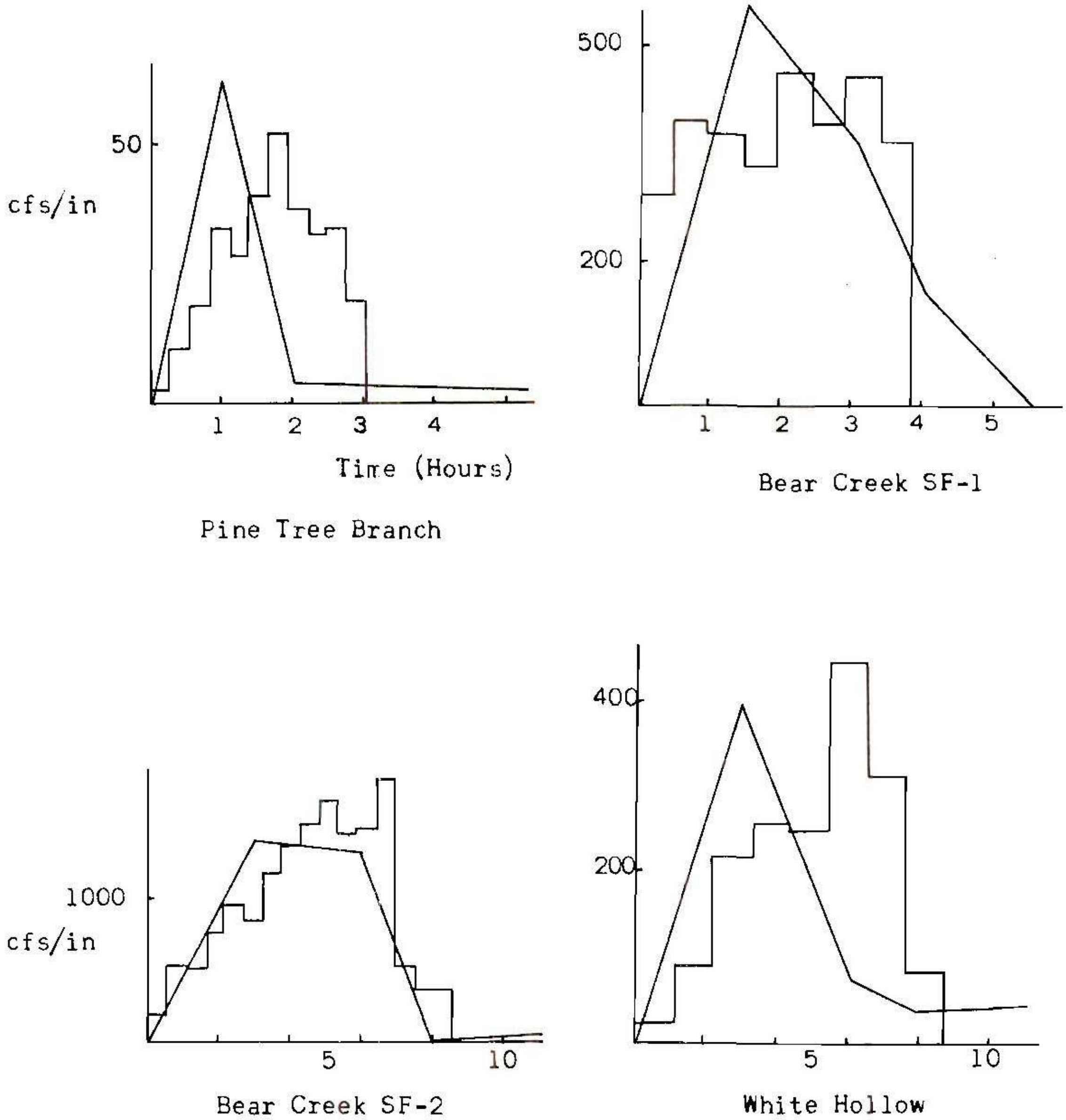


Figure 11. Characteristic Curves and Time-Area Histograms.

hydrograph peak flows increase (and times to peak decrease) with increases in rainfall intensity. The peak flow -- rainfall intensity relationship has been partly substantiated by this study.

An analysis of average unit responses revealed that two of the four watersheds appear to have higher peak flows for higher average intensities of rainfall. This relationship can be noted in Figure 12. This illustration is a plot of average storm intensities and average unit-hydrograph peak flows. Points representing storms on Pine Tree Branch and Bear Creek SF-2 watersheds show a tendency for the unit hydrographs to have higher peak flows for higher average intensities of rainfall.

An attempt to correlate individual unit-hydrograph peak flows with intensities of rainfall within storms was not successful. The model is structured in such a way that if rainfall intensity does actually effect the watershed response during a storm, the effect is not made evident. Rainfall intensity would have to be correlated to the routing parameters, flow and change in flow, to show its effect through this model. There was no such correlation in the storms used in this study.

Limitations of the Technique

A measure of the success of a hydrograph analysis technique is how reliably the technique produces results which are hydrologically reasonable. It is not sufficient that meaningful results can be obtained from some storms. Meaningful results should be obtained from most of the events analyzed.

This study was not as successful as the earlier work with this

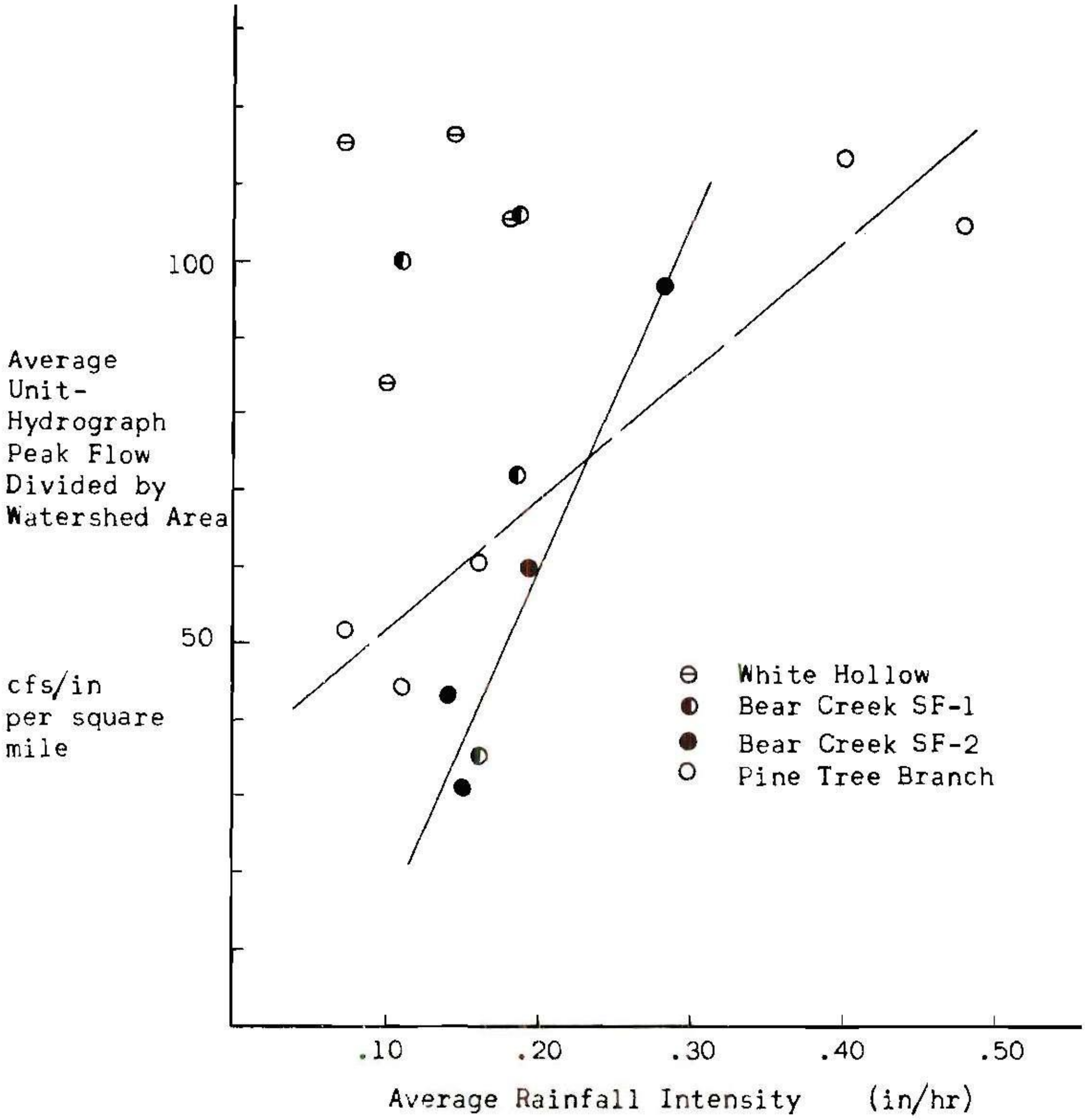


Figure 12. Average Unit Hydrograph Peak Flows vs. Average Rainfall Intensities.

model. Several problems were encountered which were not evident in the earlier application of this model. The problems were computational in nature. A few of the difficulties were severe and prevented the use of some events in the analysis. However, most of the problems were minor and, although significant, did not prevent use of the data.

Tables 3 and 4 list the problems encountered in this study. They were either related to the optimizing technique (Table 3) or involved unit responses which were too large or too small (Table 4).

Optimizing Technique

The iterative process of non-linear least squares and components regression was used to make corrections to the model parameters to improve the fit of the re-created hydrograph to the observed hydrograph. An artificial storm hydrograph obtained by input values of the parameters is analyzed to find what corrections need to be made to improve the fit of this hydrograph with the observed event. These corrections are made to the parameter coefficients and a new predicted hydrograph is computed. The process is repeated. Normally, the corrections to the model parameters become smaller with each iteration until the corrections no longer significantly affect the parameter coefficients. A problem exists if the corrections do not become insignificant after a number of iterations.

No Results. In a few cases the corrections made by the optimizing technique prevented the computer from producing any results. The corrections caused arithmetic computations to exceed the ability of the computer. The problem occurred in computing ordinates of the routing equation,

Table 3. Failures of the Optimizing Routine

Watershed	Date of Storm	Non-Convergence	Computer Failure (No Results)
White Hollow	11-25-64	X	
	12-11-64		
	3- 8-56	X	X
	12- 7-57		
	3- 4-66*	X	
	10-18-66*	X	X
Bear Creek SF-1	4- 7-64		
	2-10-65		
	3-25-64	X	
	2-11-64		
	3-14-64		
Bear Creek SF-2	4- 7-64	X	
	2-10-65	X	
	3-25-64		
	3-14-64		
Pine Tree Branch	3- 8-60		
	10-27-64		
	11-17-58		
	4-10-62		
	4-15-58	X	
	1-21-59*	X	
	3-16-63*		
	11-13-57*		X

*These events not retained in analysis.

Table 4. Errors in Volumetric Continuity

Watershed and Date of Storm	Per Cent Error in Size of Characteristic Curve	Per Cent Error in Size of Smallest Unit Response	Per Cent Error in Size of Largest Unit Response	Negative Unit Response
White Hollow				
11-25-64	2	-90	-3	
12-11-64	-9	-65	-2	
3- 8-56	5	-8	8	Yes
12- 7-57	-13	-84	-14	Yes
Bear Creek SF-1				
4- 7-64	-1	-13	-1	
2-10-65	-7	-25	-1	
3-25-64	-4	-69	6	
2-11-64	-7	-14	1	
Bear Creek SF-2				
4- 7-64	0	-4	0	Yes
2-10-65	-6	-24	2	
3-25-64	-3	-31	-3	
3-14-64	6	-70	19	
Pine Tree Branch				
3-8-60	-20	-55	-13	
10-27-64	-3	-4	-3	
11-17-58	-3	-15	-4	
4-10-62	-10	-10	-10	
4-15-58	-11	-48	-3	

$$q = \frac{1}{K} e^{-t/K} \quad . \tag{7}$$

The optimizing technique would make K very small, and consequently, the exponent, t/K, would become too large for the computer.

The optimizing technique appeared to be working incorrectly whenever this problem occurred. The corrections which make K very small were unreasonable when compared with the corresponding corrections in preceding iterations.

This problem occurred twice with the White Hollow data and once with the Pine Tree data. The problem was eliminated in two of these cases by relocating the angle points of the characteristic function and by using a larger time increment.

Non-convergence. Another problem involving the optimizing routine was non-convergence of the parameters. The corrections made to the parameters at each iteration would not become smaller as the procedure progressed. Instead, the corrections to one or more coefficients would be alternately positive and negative and would therefore cause the coefficients to oscillate across some band of values. Fortunately, this band of oscillation was sufficiently small in most cases so that fairly accurate approximations of the parameter coefficients could be made.

Although lack of convergence did not usually prevent use of events in the analysis, non-convergence was still a significant problem. This problem is significant for two reasons. First, non-convergence may indicate that the optimizing technique is not working as efficiently as it should. Second, non-convergence requires a large amount of costly computer time.

There were nine non-convergent events in the data analyzed. Table 3 shows that these non-convergent events are distributed quite evenly among the four watersheds. Six of the non-convergent events were retained in the analysis.

Volumetric Continuity

The other class of problems encountered in this study involved volumetric continuity. The volumetric continuity problem refers to the characteristic curve or the unit responses being larger or smaller than the size required for the correct input-output relationship.

The model was designed so that all unit responses would have the same area. The routing function has a form such that all unit responses derived from it should have the same area and that this area should be equal to the area under the characteristic function. Furthermore, it is assumed that the fitting technique will make the area of the characteristic function equivalent to a volume of water one inch deep over the watershed area.

These procedures to maintain volumetric continuity did not work as well in this study as they should have. Table 4 indicates some of the problems encountered pertaining to continuity. It should be noted that several of the characteristic curves are more than 10 per cent larger or smaller than their correct size. Also, a number of events have unit responses whose areas are more than 25 per cent larger or smaller than the area of the corresponding characteristic curve. The magnitude of these errors listed in Table 4 indicates that a number of events failed to follow volumetric continuity requirements closely.

Some variation in the sizes of characteristic curves and unit responses is expected. However, the degree of variability witnessed in this study is a cause for concern.

Several factors could cause these variations in characteristic curves and unit response curves. Differences in the total areas of the observed and predicted hydrographs could result in an error in the area of the characteristic curve. Routing functions which contain areas larger or smaller than unity could cause errors in the size of the unit responses and indirectly could cause errors in the area under the characteristic curves.

Hydrograph Tails. Volumetric continuity is maintained only if the area under the total predicted hydrograph is equal to the area under the total observed hydrograph. The optimizing technique, however, does not operate to establish a predicted hydrograph that is the same geometric size as the observed hydrograph. The technique operates to bring about the best fit of the ordinates of the observed hydrograph. Consequently, the area under the predicted ordinates usually approximates the area under the observed ordinates. However, the areas under the hydrograph tails, those portions of the hydrographs after the last ordinates, may be greatly different. Thus, the areas under the predicted ordinates may be exactly equal to the area under the observed ordinates but because of unequal tails the total areas of the observed and predicted hydrographs may be slightly different.

Several of the predicted hydrographs had tails which appeared to be larger or smaller than the tails of the corresponding observed hydrographs. These differences in the tails could account for differences in the total areas of the hydrographs. In turn, the result of these differences in total areas could be errors in the sizes of characteristic and unit response curves. Normally these errors would be small, but under some circumstances they may be significant.

Routing Function. The form of the routing equation,

$$q = \frac{1}{K} e^{-t/K} \tag{7}$$

is such that the area under this curve from zero to infinity is always unity. Theoretically, convolution of the characteristic curve with this unit-area routing curve should always produce unit hydrographs equal in area to the characteristic curve. However, Table 4 shows that a few storms have hydrographs that are quite different in size from the characteristic curve.

Unit hydrographs which are larger than their corresponding characteristic curves are caused by using discrete values from the routing function. The use of discrete values from the routing equation is essentially replacing the routing curve with a histogram determined by ordinates of the routing curve. The area of such a histogram, in most cases, is approximately the same as the area under the routing curve. However, in cases where the routing curve is initially steep, the area of the histogram is greater than the area of the routing curve. Consequently, the characteristic curve is convolved with a shape whose area is greater than unity and the resulting hydrograph is therefore greater in size than the characteristic curve.

In several cases the problem caused by using ordinates of the routing function was eliminated by using integrals of the routing function.

The other extreme, unit responses that are too small, is caused by uncommonly large storage coefficients. When the storage coefficient becomes large, the area under the routing curve becomes stretched out in time. This area may be so extended that less than half of the area under the routing curve is accounted for in fifty or sixty time units.

If the resulting small and extended hydrograph represents a flow condition which might occur, then the hydrograph and the routing curve are justified. However, some of the routing curves and the resulting hydrographs appear quite unreasonable. For example, one event produced a storage coefficient of over 500 hours. A storage coefficient this large appears unreasonable.

Another problem caused by the routing function was negative unit-response functions. Negative unit responses (see Table 4) occurred whenever the storage coefficient, K, became less than zero. In many cases negative responses occurred for periods of little rainfall. In these cases, the event was still used in the study.

Significance of Continuity Error. The problem of under-sized or over-sized unit responses caused by the routing function being either too large, too small, or negative, is a cause for concern. Although substantial errors in the unit responses only prevented use of two events in the analysis, these errors are significant for two reasons. First, the unit hydrographs are unreasonable. Unit responses which contain more or less volume of runoff than is created by one inch of effective rain over the watershed area are obviously incorrect. The negative response is, of course, impossible. The second reason for concern is that routing curve variations have an effect on the size and shape of the characteristic curve. If the routing curves are too large or too small, the optimizing technique compensates by making the characteristic curve larger or smaller. The variation this causes in the characteristic curves from storm to storm may obscure variations in the characteristic curve caused by significant factors.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

Non-linearity of Response

The results of this study support the belief that a non-linear relationship does exist between rainfall and runoff for the watersheds considered. The results have shown not only that there is a variation in watershed response between storms but also that there are variations in response within single events.

In addition to showing variations in the watershed response within storms, the study has shown that these variations are erratic. For example, two storms may appear similar in many ways but the analysis will reveal two different types of response variations. Unit hydrographs from one storm may become more peaked as the event progresses, whereas the unit hydrographs from the other event may become less peaked as the storm progresses.

The conclusion drawn from these response variations is that the watershed is not only a non-linear system but also, to some extent, a different non-linear system for each storm event. As Amorocho and Orlob [1961, p. 4] have stated, "...the degree of nonlinearity may be high or low in individual basins and may become more or less apparent at different times during the history of a hydrologic episode."

Variations in Unit-Hydrograph Peak Flows

This study partially substantiates the findings of earlier studies [Minshall, 1960; Singh, 1964; Prasad, 1967] that have linked unit-hydrograph peak flows to rainfall intensities. Average unit-hydrograph peak flows are shown to increase with increases in average storm rainfall intensities on two of the watersheds studied. The possibility that a relationship exists between individual storm period intensities and unit responses should not be ignored although this model failed to reveal such a relationship.

The Characteristic Function

The variability of the results and other problems encountered in the analysis make it difficult to conclude how watershed characteristics influence the model parameters. However, one conclusion can be made about the characteristic curve. This study has shown that the characteristic curve is not a reasonable approximation of the time-area curve which several hydrologists have routed to form hydrographs. Comparisons of characteristic curves and time-area curves reveal that the two shapes are different.

The watershed topography must undoubtedly affect the shape of the characteristic function. The shape of the watershed alone, however, cannot define the characteristic curve. As Snyder et al. [1969] have suggested, the characteristic curve represents the potential of the watershed for runoff production. The area and shape of the portion of the watershed which produces runoff may be greatly different from the physical area and shape of the entire watershed. Therefore, the characteristic curve, which represents runoff production, may be different from the time-area curve, which represents actual watershed topography.

The Routing Function

The concept of routing an area to create the unit hydrograph is shown to work well on the watersheds considered. The goodness of the fits of the observed events attests the flexibility of the technique.

However, there is a possibility that for some events the technique may be merely a numeric success. The technique may be producing results which cause a good fit of the observed hydrograph but do not closely reflect the actual hydrology.

The routing function should reflect actual flow patterns. The outflow hydraulics, which determine the routing function, must either determine these flow patterns or be a good index of those factors which do determine the flow patterns. There is reason to believe that channel hydraulics, the flow and change in flow, are not the controlling factors in determining the hydrograph shapes for small watersheds. For example, the unit responses for the Pine Tree Branch watershed varied greatly for some storms which produced relatively small flows. The variation in response that could be attributed to channel hydraulics could hardly account for these large observed variations.

If channel hydraulics are not the controlling factor in determining the hydrograph variations, then one must ask whether channel hydraulics are a good index to those factors which do determine the hydrograph shape. In a watershed there are many factors which change during a storm event and which could affect the hydrograph shape. The changes in these factors may or may not be correlated to the outflow hydraulics. For example, if soil moisture affects the response, the flow of the stream at the gage might serve as an index of the changing soil moisture. However, there may be better measures of watershed wetness than flow. Future studies should determine if there are better measures of the factors which cause variations in the watershed response.

The time rate of change in flow, the other hydraulic measure in the routing function, was included to represent unsteady flow effects witnessed in natural channels [Prasad, 1967]. Unsteady flow effects refer to differences in the storage-outflow relationship for the rising and falling limbs of the hydrograph. Unsteady flow effects are probably not significant in the small watersheds of this study. The optimizing technique gives the flow rate-of-change coefficient values which possibly exaggerate the importance of unsteady flow effects in determining unit-hydrograph shapes.

Recommendations

The problems and difficulties encountered in this study should help future investigators to perfect the technique. Several of the problems in this study might be easily solved. Others may require a great deal of work.

Volumetric continuity, the problem of deriving characteristic curves and unit response curves which enclose an area representing the watershed area-inch of runoff, can be perfected by constraining the model. Angle points of the characteristic curve are already constrained to have only positive values by a subroutine in the computer program. It would not be difficult to further limit the model. The greatest advance toward preserving volume continuity would be to limit the value of the storage coefficient to rational values. Limiting the storage coefficient to be a positive number less than 100 time units is a constant which should be investigated.

For a great number of events stable values of the analysis parameters were never obtained. Almost half of the events were non-convergent. The high incidence of non-convergence may indicate that the optimizing technique is not operating as efficiently as it should for this model. Even though the earlier study with this model encountered no problem of non-convergence, the high incidence of non-convergence in this study is a cause for concern. Future users of this method of hydrograph analysis should be aware of this problem and alert to find specifically what causes poor convergence of the analysis parameters.

A P P E N D I C E S

APPENDIX A

THE TIME-AREA CURVE

Ideally, the time-area curve is derived from information about the actual times of flow of runoff to the gage from each area of the watershed. Times of flow are either computed directly from measured stream velocities or computed from velocities estimated from considerations of channel and surface characteristics.

Unfortunately, the velocities of flow within the watersheds were not known. Furthermore, detailed information about channel roughness and waterway geometry was also unavailable. Lack of firm knowledge of the flow velocities made rigorous determination of the time-area diagram impossible.

Another approach to defining the time-area curve is to assume the velocity of flow constant for the entire length of the stream. This is a reasonable assumption and has been used by at least one hydrologist [Singh, 1964]. This assumption is based upon the premise that the factors which determine flow velocity vary along the stream length in such a way that the resultant velocity is fairly uniform for all points along the stream. The steeper gradients and the greater hydraulic roughness of the channels in the more remote portions of the watershed produce roughly balanced, opposite effects. Consequently, the flow velocities in the upper reaches of the streams are not greatly different from the velocities in reaches closer to the gage.

The channels of the watersheds of this study were assumed to have a constant velocity of flow throughout their lengths. This assumption greatly aided the computation of the time-area diagrams.

By assuming that water flows at the same velocity at all points along the streams, the determination of times of travel is based simply upon distances from the gage to points along the stream. However, there is still a problem in defining the time scale of the time-area curve. By assuming velocity to be uniform along the stream, one has created a distance-area histogram, but a knowledge of stream velocity is still needed to convert the distance-area diagram to a time-area diagram. The lack of precise velocity information, that prevented a rigorous determination of discrete times of travel, also prevented the determination of any average time of travel or total travel time. Furthermore, empirical methods of estimating the total travel time, or time of concentration, such as,

$$t_c = 0.6\, t_p \tag{9}$$

assume linearity of response and are consequently not applicable.

The method chosen to define the time base of the time-area curve presupposes that the time-area curve is similar to the characteristic curve. The time base of the time-area curve was set so that the first moment of the time-area curve is equal to the first moment of the characteristic curve. In other words, the time-area curve was scaled in time so that the times to the centroids of the time-area curve and characteristic curve are the same. Though not completely justifiable, this procedure does offer a means of comparing shapes of the two curves.

APPENDIX B

ROUTING COEFFICIENTS AND CORRELATION COEFFICIENTS FOR EACH STORM

Table 4. Values of the Routing Coefficient for Each Storm Period

Watershed Date Storm Period	Routing Coefficient K (in hours)	Watershed Date Storm Period	Routing Coefficient K (in hours)
White Hollow		White Hollow (cont.)	
11-25-64		3-8-56	
1	27.08	1	13.64
2	26.35	2	17.08
3	26.67	3	18.95
4	26.90	4	20.73
5	27.27	5	32.54
6	27.80	6	-1096.01
7	28.10		
8	26.82		
9	30.09	12-7-57	
10	31.34		
11	40.35	1	4.44
12	72.49	2	5.94
13	104.18	3	5.52
14	237.86	4	7.72
15	577.58	5	5.90
16	599.05	6	8.78
17	487.51	7	18.48
18	372.28	8	51.54
19	381.08	9	-18.86
		10	-90.36
		11	62.84
12-11-64		12	19.14
		13	11.72
1	6.74	14	10.08
2	10.08	15	8.60
3	8.76	16	8.02
4	8.22		
5	8.49		
6	8.80	Bear Creek SF-1	
7	8.65	2-10-65	
8	20.82		
9	20.46	1	3.75
10	27.75	2	3.91
11	158.66	3	3.89
12	29.30	4	4.04
13	130.89	5	4.20

(Continued)

Table 4. Values of the Routing Coefficient for Each Storm Period (Continued)

Watershed Date Storm Period	Routing Coefficient K (in hours)	Watershed Date Storm Period	Routing Coefficient K (in hours)
Bear Creek SF-1		Bear Creek SF-1	
2-10-65 (cont.)		4-7-64 (cont.)	
6	4.55	11	3.38
7	4.79	12	3.36
8	5.17	13	3.24
9	5.43	14	3.32
10	5.74	15	3.09
11	6.01	16	3.16
12	6.26	17	3.36
13	6.66	18	3.34
14	7.53	19	3.34
15	7.22	20	3.33
		21	3.44
		22	3.44
2-11-65		23	3.32
		24	3.44
1	2.86	25	3.21
2	3.10	26	2.52
3	3.04	27	1.27
4	3.15	28	1.32
5	3.25		
6	3.35		
7	3.53	3-25-64	
8	3.80		
9	4.52	1	52.96
		2	31.02
		3	50.11
4-7-64		4	30.02
		5	47.55
1	3.85	6	29.08
2	3.62	7	28.20
3	3.75	8	27.36
4	3.61	9	12.87
5	3.47	10	10.49
6	3.58	11	9.92
7	3.57	12	7.52
8	3.44	13	8.80
9	3.41	14	9.46
10	3.52	15	12.01

(Continued)

Table 4. Values of the Routing Coefficient for Each Storm Period (Continued)

Watershed Date Storm Period	Routing Coefficient K (in hours)	Watershed Date Storm Period	Routing Coefficient K (in hours)
Bear Creek SF-1		Bear Creek SF-2	
3-25-64 (cont.)		3-25-64 (cont.)	
16	13.87	7	4.86
17	13.67	8	3.68
18	13.47	9	2.99
19	16.14	10	3.12
20	16.14	11	6.60
21	16.14	12	33.57
22	16.14	13	12.74
23	16.14	14	7.45
24	16.14	15	7.20
25	13.28	16	2.34
26	7.70	17	2.27
27	7.40	18	1.75
28	7.11		
29	6.85		
30	4.59	3-14-64	
31	2.49		
32	2.34	1	4.59
33	3.27	2	4.75
34	4.74	3	4.68
35	9.20	4	4.73
36	41.30	5	4.80
37	24.90	6	4.86
38	54.99	7	4.92
39	20.59	8	5.00
40	14.53	9	5.12
41	10.96	10	5.07
42	13.20	11	5.06
		12	5.02
		13	4.96
Bear Creek SF-2		14	4.99
3-25-64		15	5.01
		16	5.12
1	15.52	17	5.55
2	11.60	18	6.00
3	12.59	19	9.98
4	10.54	20	121.76
5	9.30	21	29.35
6	6.67	22	33.96

(Continued)

Table 4. Values of the Routing Coefficient for Each Storm Period (Continued)

Watershed Date Storm Period	Routing Coefficient K (in hours)	Watershed Date Storm Period	Routing Coefficient K (in hours)
Bear Creek SF-2 (cont.)		Bear Creek SF-2	
2-10-65		4-7-64	
1	5.16	5	2.45
2	4.37	6	2.55
3	4.87	7	2.61
4	5.03	8	2.79
5	4.24	9	2.96
6	4.69	10	3.08
7	7.53	11	3.50
8	35.14	12	3.80
9	-44.69	13	6.53
10	-14.32	14	6.20
11	-10.21	15	8.70
12	-9.10	16	-43.12
13	-10.84		
14	-88.97		
15	-9.54	Pine Tree Branch	
16	79.64	10-27-64	
17	22.81		
18	15.74	1	5.58
19	12.92	2	3.94
20	11.09	3	3.96
21	10.05	4	2.65
22	9.36		
23	8.79		
24	8.23	3-8-60	
25	7.41		
26	6.86		
27	6.86	1	5.91
28	5.84	2	9.02
29	5.94	3	8.03
30	9.33	4	8.41
		5	37.18
		6	-25.89
4-7-64			
1	2.20	4-10-62	
2	2.42		
3	2.35	1	3.07
4	2.42	2	1.73

(Continued)

Table 4. Values of the Routing Coefficient for Each Storm Period (Continued)

Watershed Date Storm Period	Routing Coefficient K (in hours)
Pine Tree Branch	
4-10-62 (cont.)	
3	1.95
4	2.31
5	2.23
6	1.91
7	.98
8	.71
11-17-58	
1	6.20
2	6.78
3	6.63
4	6.78
5	7.20
6	7.35
7	8.90
8	9.38
9	11.73
10	10.77
11	9.67
4-15-58	
1	5.66
2	8.57
3	9.02
4	10.70
5	13.43
6	16.08
7	18.83
8	31.63

Table 5. Correlation Coefficients

Watershed	Date of Storm	Correlation Coefficient
White Hollow	11-25-64	0.998
	12-11-64	0.972
	3- 8 -56	0.984
	12- 7 -57	0.894
Bear Creek SF-1	2-10-65	0.986
	2-11-65	0.995
	4- 7 -64	0.995
	3-25-64	0.989
Bear Creek SF-2	3-25-64	0.977
	3-14-64	0.986
	2-10-65	-
	4- 7 -64	0.981
Pine Tree Branch	10-27-64	0.993
	3- 8 -60	0.907
	4-10-62	0.972
	11-17-58	0.994
	4-15-58	0.971

APPENDIX C

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